Syllabus Fall, 2011

MATH 523, SCHC 411B, BIOL 763

Mathematical Modeling of Population Biology

Professors.

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Text and computing materials. Ted J. Case, *An Illustrated Guide to Theoretical Ecology*, Oxford University Press, 2000. We will try to work in some models from other parts of biology, although, truth to tell, it is a lot easier to understand population counts or densities than concentrations or moles (the chemical kind), etc. This text contains some genuine biology, but you don't need any specialized knowledge of ecology to use it. Computational work will mostly be done with Maple or R, available in the classroom/lab (personal copies can be purchased or downloaded for free). Research articles will be provided or web access given.

Prerequisite. You must have earned a grade of C or better in MATH 122, 141, or an equivalent calculus course to be approved by the instructor. This pre-requisite is essential. Some background in ecology will be useful, but is not essential.

Overview. This is a team taught course with biologist/ecologist Dave Wethey of the Department of Biological Sciences. The course is not intended to be merely a collection of mathematical tools that you might someday, maybe, use in biology; rather it is an introduction to a way of thinking about biology that uses mathematics to create a conceptual and unified approach. Model building and model analysis, both quantitative and qualitative, will be as important as model solving. We develop a number of mathematical models for single and multiple species systems to describe population dynamics (both continuous and discrete) over time. Themes include populations with age structure, limitations on growth, harvesting, competition, and predator-prey interactions. We are especially interested in predictions of long term behavior, as this is what one expects to see in the field: equilibrium states and oscillations, for example, and whether these are stable (small perturbations have little effect) or unstable. We use qualitative tools, especially graphical ones, for conceptual analysis, and Maple or R for computations and simulations.

You will possibly find that this course is very different from other math courses that you have taken. We will be less concerned with the mechanical aspects of computation, and more concerned with why we want to do these calculations. We form a mathematical model of a changing real world situation, such as population growth, use a variety of methods to analyze it, and then interpret our calculated results in the context of the original problem.

We will analyze problems by using a blend of verbal, pictorial, numerical, graphical, and analytic methods (manipulation of formulas). Modeling is more comprehensive than problem solving; we will learn how models are built, as well as how to "read" them. One of the goals of the course is to enable you to look at scary formulas and to be able to dissect them into small and comprehensible pieces, so that even if you don't "do the math" yourself, you will be able to follow what has been done. Finally, in the real world, solutions must be communicated effectively, both in writing and orally, and you will get some practice doing this.

Learning Outcomes. Students will be able to read original biological modeling literature and be able to analyze it critically. They will be able to translate verbal descriptions of a process into mathematical language and vice versa. They will be able to recognize standard models, and be able to modify them and run simulations using Maple or R templates provided by the instructors, and then be able to interpret the results in a biological setting. Along the way they will master the nomenclature of the field, and demonstrate understanding of terms such as intrinsic rate of growth, initial value problem, equilibrium, cycle, bifurcation, chaos, consumer functional response, eigenvalue-eigenvector pair, phase plane, and parameter space.

Our goal in short is to make the theoretical and modeling literature in biology accessible so that the students may incorporate it into their future research projects.

Course content. By the end of the course, students will understand the concepts of and be able to solve problems drawn from biological modeling with differential and difference equations; techniques of model modifications; analytic, numerical, and graphical solution methods; equilibria, stability, and long-term system behavior; geometric series; vectors, matrices, eigenvalues, and eigenvectors, with applications principally to population dynamics and compartment models.

We begin with an introduction to the idea of modeling a system (why do it?), how to build models starting with really simple ones and gradually building up to more complex ones, and how to interpret models that you can pull off the shelf. Models come in two flavors: discrete and continuous; we will explore how these are similar, how they are different, and why one might chose one over the other. The discrete ones are usually called difference equations; the continuous ones are called differential equations (DE's) and these you have already been solving in your calculus classes! Every time that you concluded something about a function, say $P(t)$, population as a function of time, from information about its rate of change $P'(t)$ or $\frac{dP}{dt}$ or even $\dot{P}$, you were actually solving a DE. One of our main tools will be the study of the equilibria (stationary values) of a model, what these are, how to compute them, how to determine if they are stable or not, and how they are related to the long term behavior of a system. Put crudely, we want to use models make predictions such as whether a population (or an epidemic) will boom (turn into a pandemic), go extinct (die out), or fluctuate around a certain level. Next we will learn a little bit about matrices and vectors, emphasizing the intuitive geometric point of view, and then go on to selected topics involving multi-species or metapopulation models that use these basic tools. In terms of the text we will cover chapters 1, 3, 5, 10, 12, 13, 14 and 16 fairly thoroughly, and chapters 2, 4, 6, 7, 8, 11, 15 and other topics much more selectively. All along we will work with tables of data, or verbal descriptions of problems, to build our models, and we will use our calculators to make educated guesses about the **qualitative behavior** of the solutions You will be expected to **gradually recognize for yourself** when the use of technology is appropriate, and when hand computation and exact algebra or calculus is called for.

Grades. The final grade is based upon the written and oral presentation of two group projects, based on critical analyses of published articls together with extensions of the "What if?" flavor, and a written final exam. A 15-minute group oral presentation will precede the submission of the written report; you will be expected to incorporate feedback from the presentation into the written version. Each project will coount for 1/3 of your final grade, as will the final exam. Letter grades will be based upon the accuracy, completeness, and quality of presentation of the work.

Collaboration. One of the goals of this course is to learn how to communicate ideas. You will be expected to work with one another in class and we expect that you will do so on the projects. However, you will have to take the final exam individually, so don't get too dependent upon one another. According to the USC Student Handbook code of student academic responsibility, "**the first law of academic life is intellectual honesty.**" I expect this of all of you. If you are ever in the least bit uncertain about the ground-rules, ask for clarification!

Attendance. Regular attendance is crucial for success in this course. This class has 29 meetings; university policy states that if more than 10% of the meetings are missed, whether excused or unexcused, then the instructor may impose a penalty. We intend that this be a very rich and varied class, often with non-lecture activities. If you feel that a class is nothing more than a series of exams and some assignments to be turned in, with attendance optional, then this is not the class for you. If you miss 4 or more class sessions, we will lower your grade by half a grade point (from an A to a B+, or a C+ to a C, for example), and if you miss 6 or more classes (that's three weeks!), your grade will drop by a full grade point.

Having fun doing math. We hope that the model building and analysis, the various verbal, numerical and graphical approaches, and the general de-emphasis on rote problem solving and formula memorization (there is some, but not much), will be stimulating. In many cases there will be more "thinking about the math" than "doing the math." This might be a little unsettling at first, because it means that you won't be doing the same problem over and over again, but will be called upon to think most problems through from scratch. We think this material is really beautiful, and we hope you will too.