MATH 520 **Spring, 2012** **Quiz #5** **Name:**____________________

1. Let $A = \begin{bmatrix} 6 & -3 \\ 2 & 1 \end{bmatrix}$. Write the general solution for $\mathbf{x}' = A\mathbf{x}$.

$$\begin{vmatrix} 6-r & -3 \\ 2 & 1-r \end{vmatrix} = 6 - 7r + r^2 + 6 = r^2 - 7r + 12$$
$$= (r-3)(r-4)$$

$$r=3 \quad \left[\begin{array}{cc|c} 3 & -3 & 0 \\ 2 & -2 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right] \quad \vec{\xi}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

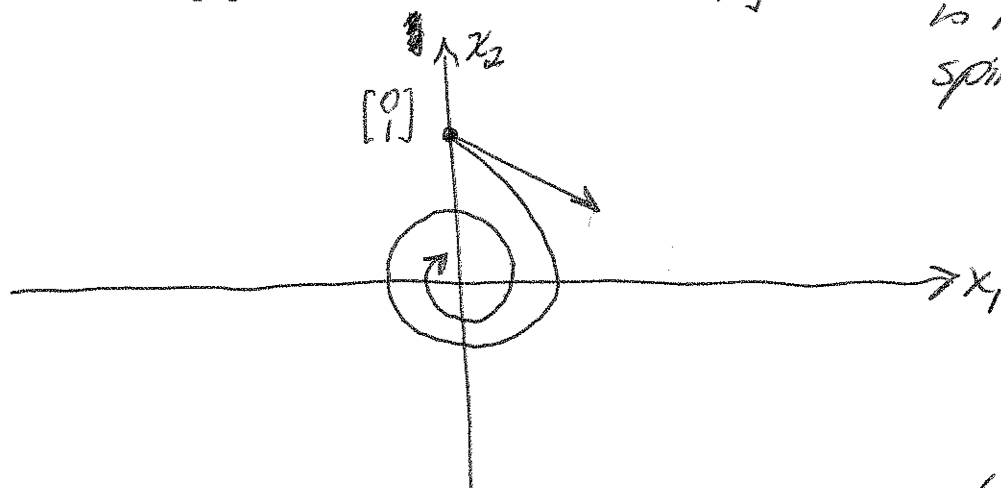
$$r=4 \quad \left[\begin{array}{cc|c} 2 & -3 & 0 \\ 2 & -3 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{cc|c} 1 & -3/2 & 0 \\ 0 & 0 & 0 \end{array}\right] \quad \vec{\xi}_2 = \begin{bmatrix} 3/2 \\ 1 \end{bmatrix} \text{ or } \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$\vec{x} = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{3t} + c_2 \begin{bmatrix} 3 \\ 2 \end{bmatrix} e^{4t}$$

2. The matrix $A = \begin{bmatrix} -3 & 2 \\ -1 & -1 \end{bmatrix}$ has eigenvalues $r = -2 \pm i$. Sketch the trajectory from the point $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

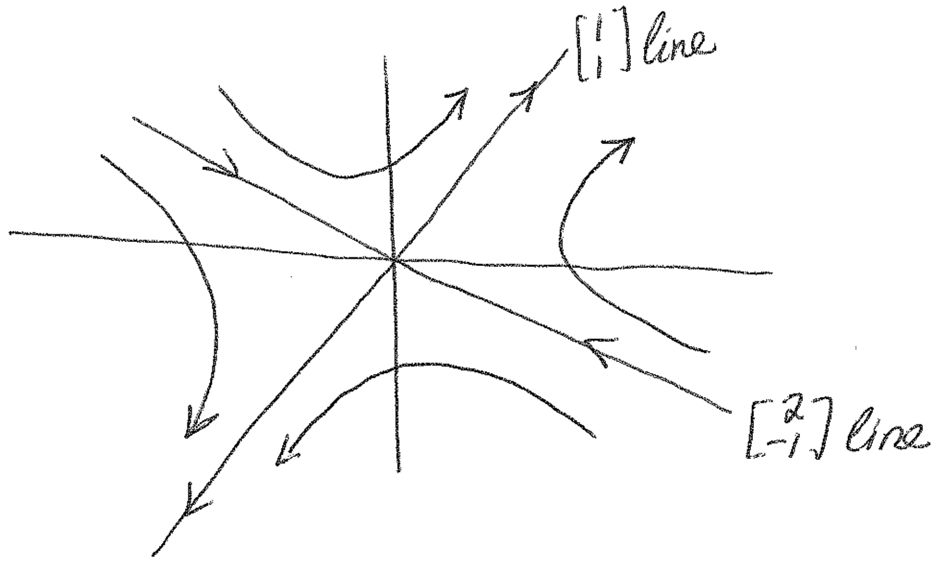
$$A \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

Real part of r is negative so spiralling in



(over →)

3. The matrix A has eigenvalue, eigenvector pairs 3, $\begin{bmatrix}1\\1\end{bmatrix}$ and -2, $\begin{bmatrix}2\\-1\end{bmatrix}$. Write the general solution of $\mathbf{x}' = A\mathbf{x}$, and sketch the phase plane around the origin.



$$\vec{x} = c_1 \begin{bmatrix}1\\1\end{bmatrix} e^{3t} + c_2 \begin{bmatrix}2\\-1\end{bmatrix} e^{-2t}$$