

1. Find the general solution of $y'' + y = 10e^{2t}$ (note that the coefficient of y' is zero).

$$r^2 + 1 = 0$$

$$r^2 = -1$$

$$r = \pm i$$

Solve $y'' + y = 0$ first.

$y = e^{it}, y = e^{-it}$ complex fund. solns.

$y_1 = \cos x, y_2 = \sin x$ real fund.

Seek particular solution $y_p = Ae^{2t}$ solutions

$$y_p' = 2Ae^{2t}$$

$$y_p'' = 4Ae^{2t}$$

$$y_p'' + y_p = 5Ae^{2t} = 10e^{2t}$$

$$A = 2$$

General solution $y = c_1 \cos t + c_2 \sin t + 2e^{2t} = y$

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2. We have $y' = y(10-y)(y-1)$. Find the equilibria, the phase line, and stability of each equilibrium point. Draw a comprehensive graph showing typical solution curves.

$$y^* = 0, 10, 1$$

$$y > 10 \quad y' < 0$$

$$1 < y < 10 \quad y' > 0$$

$$0 < y < 1 \quad y' < 0$$

$$y < 0 \quad y' > 0$$

