

1. Find the general solution of the linear first order equation $y' + 2ty = 2te^{-t^2}$.
What is the behavior of the solution as $t \rightarrow \infty$?

$$\mu = e^{\int p(t) dt} = e^{\int 2t dt} = e^{t^2}$$

No constant of integration is needed here, just need some integrating factor

$$e^{t^2} y = \mu y = \int \mu q(t) dt = \int e^{t^2} 2t e^{-t^2} dt$$

$$= \int 2t dt$$

$$= t^2 + C$$

$$y = \frac{t^2 + C}{e^{t^2}} = e^{-t^2} (t^2 + C)$$

As $t \rightarrow \infty$, e^{t^2} dominates $t^2 + C$,
so $y \rightarrow 0$; or use L'Hopital's
Rule.