MATH 520 (Section 001)
Prof. Miller / Meade

University of South Carolina
Spring 2012

Exam 2
March 15, 2012

Name: ______________________
SS # (last 4 digits): ____________

Instructions:

1. There are a total of 4 problems on 2 pages. Check that your copy of the exam has all of the problems.
2. No electronic or other inanimate objects can be used during this exam. All questions have been designed with this in mind and should not involve unreasonable manual calculations.
3. Be sure you answer the questions that are asked.
4. You must show all of your work to receive full credit for a correct answer. Correct answers with no supporting work will be eligible for at most half-credit.
5. Your answers must be clearly labeled and written legibly on additional sheets of paper (that I will provide). Be sure each sheet contains your name and the work for each question is clearly labeled.
6. Check your work. If I see *clear evidence* that you checked your answer (when possible) and you *clearly indicate* that your answer is incorrect, you will be eligible for more points than if you had not checked your work.

| Problem | Points | Score |
|---|---|---|
| 1 | 22 | |
| 2 | 24 | |
| 3 | 30 | |
| 4 | 24 | |
| Total | 100 | |

Good Luck!

1. (22 points) Transform the second-order equation $u'' + 0.25u' + 4u = 2\cos(3t)$ into a system of two first-order equations. (Recall that it is useful to set $x_1 = u$ and $x_2 = u'$.)

2. (24 points) Let $A = \begin{bmatrix} 1 & 1 \\ 4 & -2 \end{bmatrix}$ and consider the system $\mathbf{x}' = A\mathbf{x}$.

   (a) Find fundamental solutions.

   (b) Write the general solution to $\mathbf{x}' = A\mathbf{x}$.

   (c) Show that the solutions found in (a) are independent.

   (d) Sketch the phase portrait around the origin.

3. (30 points) Consider $\mathbf{x}' = A\mathbf{x}$ when $A = \begin{bmatrix} 5 & -1 \\ 3 & 1 \end{bmatrix}$. The eigenvalue-eigenvector pairs of $A$ are 2, $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$ and 4, $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

   (a) Find the general solution of $\mathbf{x}' = A\mathbf{x}$.

   (b) Find the coefficients that produces the solution that satisfies $\mathbf{x}(0) = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$.

   (c) Describe what happens to solutions as $t \to \infty$.

   (d) Describe what happens to solutions as $t \to -\infty$.

   (e) Sketch this trajectory in the phase plane. (Make use of the direction vector at $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$.)

4. (24 points) Let $A = \begin{bmatrix} 2 & -5 \\ 1 & -2 \end{bmatrix}$. One eigenvalue-eigenvector pair is $\lambda = i$ and $\mathbf{v}_1 = \begin{bmatrix} 2+i \\ 1 \end{bmatrix}$.

   (a) Confirm (check) that $\lambda = i$ and $\mathbf{v}_1 = \begin{bmatrix} 2+i \\ 1 \end{bmatrix}$ is an eigenvalue-eigenvector pair for $A$.

   (b) What is the other eigenvalue-eigenvector pair for $A$? (no work needed)

   (c) Extract the real fundamental solutions of $\mathbf{x}' = A\mathbf{x}$ from the complex solution $\mathbf{v}_1 e^{\lambda t}$.

   (d) The vector $\begin{bmatrix} 5 \\ 2-i \end{bmatrix}$ is another eigenvector for $\lambda = i$. Is it independent or dependent from $\mathbf{v}_1$? Explain.