SCCC 411B Exam 2 Spring, 1997 Name:____________________

Instructions. There are 100 points. Answer all questions. Be sure to supply adequate explanation for your answers.

1. (35 points) The Leslie matrix for a population of New Zealand sheep is given by the following data, where F_i denotes the fecundity and P_i the probability of survival to the next age class.

age (yr)	0	1	2	3	4	5	6	7	8	9	10	11
F_i	.0	.04	.39	.47	.48	.55	.54	.50	.47	.46	.43	.42
P_i	.84	.98	.96	.95	.93	.90	.85	.79	.69	.56	.37	.0

a. Write out the upper left 4×4 corner of the matrix L.

b. If the current population consists of 100 two year olds and 100 three year olds how many individuals will be in each age class one year from now?

c. The dominant eigenvalue for L is $\lambda = 1.18$, and the corresponding eigenvector is $[.24, .17, .14, .12, .10, .08, .06, .04, .03, .02, .01, .0]$. The dominant eigenvalue for L^T is $\mu = 1.18$, and the corresponding eigenvector is $[1.0, 1.4, 1.6, 1.6, 1.5, 1.3, 1.1, 0.9, 0.8, 0.6, 0.5, 0.4]$. Over the long term does the population grow, decline, or remain stable? Why? In the long term what percent of the population is 7 years old or older? Which age class has the highest reproductive value? Is this the age class with the highest fecundity? If not, explain the apparent discrepancy.

d. Recall that the elasticity e_{ij} times percent change in L_{ij} gives percent change in λ. We have calculated that $e_{13} = 0.037$ and $e_{18} = 0.014$. Suppose the fecundity of the two year olds rises from .39 to .44 and the fecundity of the seven year olds drops from .50 to .40. Which has a larger effect on λ in terms of percent change? How do you account for this intuitively?

2. (20 points)

a. You are given the plot $N_{t+1} = F(N_t)$. Based on this graph, what is the steady state $\bar{N}$? If $N_0 = 5$, use the graph to compute N_4.

b. You are given the plots $N_{t+1} = G(N_t)$, $N_{t+2} = G(N_{t+1}) = G(G(N_t))$, and the line $N_{t+2} = N_t$. Interpret the points A, B, and C in terms of stable or unstable equilibria and cycles. If $N_0 = 1.1$, what is the long term behavior of N_t?

3. (30 points) In this problem we investigate a host-parasitoid interaction in which the hosts have a possible refuge. Let N_t denote the host population at time t and P_t the parasitoid population. In the absence of parasitoids the host population grows at a per capita rate λ. Each parasitized host yields c parasitoids in the next generation. The probability of k episodes of parasitoid attack on an unprotected host in one breeding cycle is $p(k) = e^{-aP_t}(aP_t)^k/k!$. The carrying capacity of the host population is K and a fraction b of this amount can hide in a protected refuge. All parameter values are positive.

   a. How many of the host population can hide? What fraction of the total host population is this? What fraction of the host population does this leave exposed?

   b. What is the probability that an exposed host escapes being parasitized (or, what fraction of the exposed host population escapes being parasitized)?

c. What fraction of N_t is available for reproduction, either by hiding, or by being exposed but lucky?

d. Explain how the equations given below reflect our description of the model.

$$\begin{aligned} N_{t+1} &= \lambda N_t \left(\frac{bK}{N_t} + \left(1 - \frac{bK}{N_t}\right) e^{-aP_t} \right) \\ P_{t+1} &= c N_t \left(1 - \frac{bK}{N_t}\right) (1 - e^{-aP_t}) \end{aligned}$$

e. There is a potential flaw in this model. What does it predict for very low host populations (for example, around $\frac{1}{2}bK$)?

f. With the parameter values $a = 0.2$, $b = 0.1$, $c = 1$, $K = 100$, $\lambda = 2.8$, we find that there is a steady state $\bar{N} = 29.25$, $\bar{P} = 18.81$; the corresponding Jacobian matrix has eigenvalues $0.08 \pm 0.49i$. Describe the behavior of the system near the equilibrium. In what respect is this result a bit surprising? Upon reflection how do you account for this?

4. (15 points) Give brief definitions of the following terms. Illustrative pictures might also be appropriate.
   a. non-linear difference equation

   b. Lefkowitch matrix

   c. memoryless random process