SCCC 411B Spring, 1997 Test #1 Name:____________________

1. (15 points) Briefly discuss each of the following terms. Explain in words how each concept is used in the course of designing or analyzing mathematical models. Illustrations may also be helpful.

   (a) Carrying capacity

   (b) Non-dimensional form (for rate equations)

   (c) Michaelis-Menton growth rate

2. (15 points) Consider the following model for movement of phosphorus (P) in an ecosystem. Amounts within the compartments are given in mg, and amounts indicated for

transfer are given in mg/day. Let p, h, and w indicate the amount of P in plants, herbivores, and water, respectively.

(a) Write the equations that give the rates of change p', h', and w'.

(b) Is the system in equilibrium? Explain!

(c) Now suppose we write $\frac{dp}{dt} = a_{wp}w + a_{pw}p + a_{hp}h + a_{ph}p$, where a_{xy} denotes the fraction of the amount of phosphorus in x that is being transferred to y. Give the values of the four coefficients (as fractions, decimals if you prefer); don't forget the signs ($\pm$). What are the units?

3. (20 points) Consider the three phase diagrams below.

(a) Comparing (A) and (B) only, which system exhibits greater sensitivity to initial conditions? Explain!

(b) Each system has a steady state (marked by the heavy dot). For systems (B) and (C) only, determine whether this steady state is unstable or stable; briefly explain how you can tell.

(c) Suppose the eigenvalues for the linearized system at the equilibrium are $\lambda_1 = a+bi$ and $\lambda_2 = c + di$. For system (C) only, describe a, b, c, and d in terms of being positive, negative, zero, or simply non-zero, as is most appropriate.

4. (50 points) In a predator-prey system developed by Robert May, with suitable units for prey x and predators y, we have

$$\begin{aligned} \frac{dx}{dt} &= 0.6x(1 - \frac{x}{5}) - 0.5\frac{xy}{x+1} \\ \frac{dy}{dt} &= 0.1y(1 - \frac{y}{2x}) \end{aligned}$$

(a) In the absence of predators, what does this model predict?

(b) If prey are extremely abundant, what does this model predict about the predator population in the near term?

(c) Is it possible for the predator population to decline in this model? Under what circumstances?

(d) Identify the term that gives the loss in prey population due to predation. Explain the significance of this term for very low levels of prey.

(e) (**Continuation of problem 4**) Nullclines for this system are plotted below. Indicate which one goes with $\frac{dx}{dt} = 0$ and which one with $\frac{dy}{dt} = 0$. In each of the four regions cut out by the nullclines decide whether each of x and y must be increasing or decreasing. Give an arrow that represents the net direction a trajectory must have as it passes through each region. (Suggestion: you may find it useful to decide what happens to x if an initial condition lies under the arched curve, and what happens to y if an initial condition lies above and to the left of the straight line.)

(f) When Maple computes the steady states, it finds one at $(-5.3, -10.6)$. Why did we not include this one on the graph?

(g) There is another steady state at $(0.95, 1.89)$. If the system is linearized with this point as the coordinate center, one finds a coefficient matrix with eigenvalues $\lambda = 0.01 \pm 0.19i$. Based on this information, and possibly also your answer to part (e), sketch a likely trajectory on the graph if the system begins with initial condition $(1, 2)$.