SCCC 411B **Study Guide for Exam #1** **Fall, 1995**

Our goal in this course is to get you thinking about models, not cramming information into your heads. Nevertheless there are a few things that are so fundamental that you really should memorize them. Most important are the three models that are a touchstone for everything else: so memorize the rate equations that describe exponential growth, logistic growth, and chemostat controlled growth with Michaelis-Menton kinetics (equations 16ab on page 126, or equations 19ab on page 127).

The following exercises from the EK text and the handout on vectors are similar to possible exam questions. Some you have probably already done, so you just need to review your solution.

(1) Chapter 4, problems 3, 5ad, 7abc (use everything you have learned in this problem), 29a
(2) Chapter 5, problems 10, 16abcd
(3) Chapter 6, problems 9, 31, 4 (hint: $g(N)$ is the per capita rate of growth; the net growth rate is $Ng(N)$)
(4) Vector handout, problem 6. Also be able to show how SQ transforms if you are given a 2×2 matrix A.

Here are some additional problems.

1. Suppose $\frac{dx}{dt} = y - x^2$ and $\frac{dy}{dt} = x - y^2$. Find the equations of the two nullclines and sketch them. Find all equilibrium points. Sketch arrows indicating the directions of trajectories as they cross the nullclines from one region to the next (as in Figure 5.16 or Figure 6.7). Then sketch sample trajectories. At one of the equilibrium points the linearized system has coefficient matrix $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, with eigenvalues $\lambda_1 = 1$, $\lambda_2 = -1$, and eigenvectors $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$. Which eigenvalue goes with which eigenvector? At the other equilibrium point the matrix of the linearized system is $B = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}$ with eigenvalues $\mu_1 = -3$, $\mu_2 = -1$, and the same eigenvectors as for A. Which eigenvector goes with which eigenvalue? What do the trajectories look like near each equilibrium point? Discuss the type and stability features of each. Are these conclusions consistent with the trajectories you have already drawn on the global phase diagram?

2. On page 305, problem 21 gives you the equations for an activator inhibitor system. Which variable corresponds to the activator level and which to the inhibitor level? Is it possible for the amount of inhibitor to decrease in this model, and if so, under what conditions? How does the model predict that the activator will grow if little inhibitor is present? if a lot is present? Does the system have any steady states? (This would be a nice model to try your hand on going through the whole process of linearization and local analysis–but we wouldn't put such a question on the test.)

Here are some general topics to think about.

(1) Producing equations from verbal descriptions, and verifying the consistency of equations with verbal descriptions.
(2) How to read compartment (box) diagrams and produce rate equations.
(3) Dimensional analysis (how to find the units for everything in sight). We will not ask you to non-dimensionalize a system on the exam; we would probably even stop you from trying!
(4) Extracting information from graphs, especially phase portraits. Recognizing periodicity, stability, instability, long term behavior.
(5) How to use eigenvector, eigenvalue information.

Basta! Basta!