SCCC 411B Final Exam Fall, 1995 Name:____________________

Instructions. There are 100 points. Answer all questions. Be sure to supply adequate explanation for your answers.

1. (12 points) Give brief definitions of the following terms. Illustrative pictures might also be appropriate.
   a. stable 4-cycle

   b. stable age distribution

   c. boundary condition

   d. eigenvalue

   e. traveling wave

   f. difference equation

2. (20 points) The Leslie-Lefkowitch matrix for a certain population model with classes I, II, III, and IV is $A = \begin{bmatrix} 0 & 0.5 & 4.0 & 0 \\ 0.9 & 0 & 0 & 0 \\ 0 & 0.8 & 0 & 0 \\ 0 & 0 & 0.3 & 0.2 \end{bmatrix}$

a. Does this model describe age classes or stages? Why? (Suggestion: what is the significance of the last entry in the bottom row?)

b. Which age classes/stages are active reproductives? Explain.

c. If the current population consists of 100 individuals in class II, how many individuals will be in each class two time steps from now?

d. The dominant eigenvalue for this system is $\lambda = 1.53$, and the corresponding eigenvector is $[0.51, 0.3, 0.16, 0.04]$. The dominant eigenvalue for the transposed matrix is $\mu = 1.53$, and the corresponding eigenvector is $[1.0, 1.7, 2.6, 0]$. Use this information to answer the following questions. Justify your answers! Over the long term does the population grow, decline, or remain stable? What is the long term distribution of the population in percent terms? Which age class/stage has the highest reproductive value? Is this value higher or lower than the reproductive value of a newborn–and why does this make sense from the biological point of view?

3. (15 points) Consider the following model for leaf-eating herbivores (population size H_t) browsing trees with leaf mass L_t . We assume $L_t \neq 0$, and parameters a , b , r to be positive.

$$L_{t+1} = bL_t e^{-aH_t}$$
$$H_{t+1} = H_t + rH_t(1 - \frac{H_t}{L_t})$$

a. Discuss the qualitative features of this model (for instance: if there are no herbivores, what happens? if the leaf mass increases, what happens? what does the exponential term represent? how is this model similar to ones we have examined in the course?)

b. Show that $H = \frac{\ln b}{a} = L$ is a steady state for this system. Is this equilibrium feasible? (Explain why it is reasonable to assume that $b > 1$ in this model; then what do you know about $\ln b$?)

4. (25 points) The density dependent Nicholson-Bailey host-parasitoid system proposed by Beddington *et.al.* is given by

$$N_{t+1} = N_t e^{r(1-N_t/K)-aP_t}$$
$$P_{t+1} = cN_t(1-e^{-aP_t}).$$

If $\bar{N}$ and $\bar{P}$ are the positive equilibrium values for the hosts and parasitoids, respectively, and we define $q = \bar{N}/K$, then $\bar{P} = r(1-q)/a$ and $\bar{N} = \bar{P}/(1-e^{-a\bar{P}})$. For $a = 0.2$ and $c = 1.0$, the eigenvalues of the Jacobian of this system at the point $(\bar{N}, \bar{P})$ depend on r and q as shown in the following table.

r	q	λ_1	λ_2	magnitude of dominant λ
5.0	0.9	-3.38	0.65	
3.0	0.3	$0.20+1.44i$	$0.20-1.44i$	
2.0	0.9	0.78	-0.67	
1.5	0.6	$0.41+0.71i$	$0.41-0.71i$	

Complete the table by computing the magnitudes. Then for each pair (r, q), describe the expected behavior of host and parasitoid populations starting at values close to $(\bar{N}, \bar{P})$. Supplement your verbal descriptions with sketches of sample trajectories.

5. (10 points) In this problem we examine Roughgarden's theory of r and K selection.

   a. In graphs I and II the independent variable is p, the frequency of allele A in the population. The dependent variable N is the population. In both cases the intrinsic growth rates are $r_{AA} = r_{Aa} = 0.8$ and $r_{aa} = 0.5$. Likewise in both cases $K_{AA} = 7000$ and $K_{aa} = 10000$. Initial conditions all start with small population $N = 100$, but with a variety of different values for p. For each graph, does there appear to be long term fixation of an allele, or does polymorphism persist? For each graph, how large do you estimate K_{Aa} to be in comparison to K_{AA} and K_{aa}? With respect to growth under high density conditions is there heteozygote advantage or disadvantage?

b. With axes labeled the same way as before, and initial condition $N(0) = 100$, how do you interpret the following graph? Is it possible to estimate K_{AA} and K_{aa} from this graph? If yes, do so, else explain why not.

6. (10 points) Match the verbal description (letter) of the boundary or the initial condition for the one dimensional diffusion equation $\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial x^2}$ with the mathematical description (number). One can interpret $c(x,t)$ as the concentration of a chemical at location x at time t in a thin rod of length L that is full of water, or as the temperature at that spot and time in a metal rod.

A. Zero concentration maintained at both ends.
B. Chemical present at a uniform concentration at initial time.
C. Rod is sealed (or insulated); no chemical (or heat) can pass through the ends.
D. High amount of chemical injected at the center.
E. High amount of chemical injected at one end.

1. $\frac{\partial c}{\partial x} = 0$ at $x = 0$ and $x = L$ for all t.
2. $c(x,0) = 1$ for $0 \le x \le L/100$, $c(x,0) = 0$ otherwise.
3. $c(x,0) = e^{-10(x-L/2)^2/(L^2)}$ for $0 \le x \le L$.
4. $c(x,0) = 1$ for $0 \le x \le L$.
5. $c(0,t) = c(L,t) = 0$ for all t.

A. ________ B. ________ C. ________ D. ________ E. ________

7. (8 points) Discuss the relationship between the bifurcation plot and each of the plots A–F of population as a function of time. What does K on the bifurcation plot represent in terms of the plots of $N(t)$ versus t? For each plot, supply the appropriate value (or range of values) of r.