MATH 241 Spring, 2010 Quiz #8 Name: M²

For full credit you must show sufficient work that the method of obtaining your answer is clear.

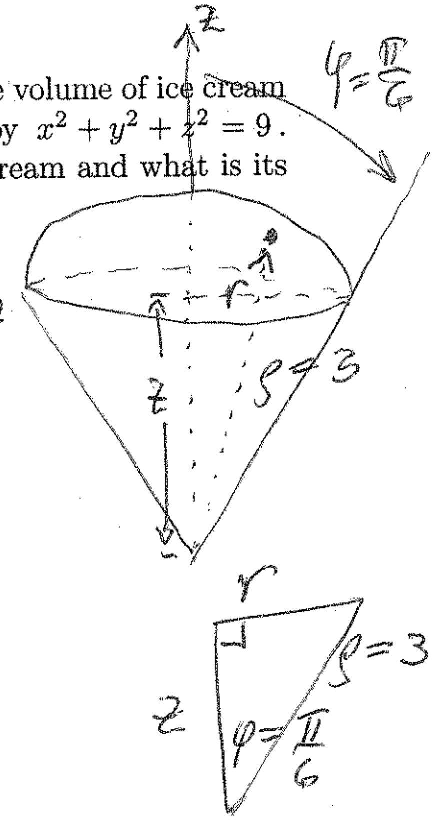
1. SET UP the integral in spherical coordinates that gives the volume of ice cream stuffed inside the cone $\varphi = \pi/6$, with top surface given by $x^2 + y^2 + z^2 = 9$. Do NOT compute. How tall is the cone without the ice cream and what is its top radius?

$\rho^2 = 9$ so $\rho = 3$

$$V = \int_0^{2\pi}\int_0^{\pi/6}\int_0^{3} \rho^2 \sin\varphi \, d\rho \, d\varphi \, d\theta$$

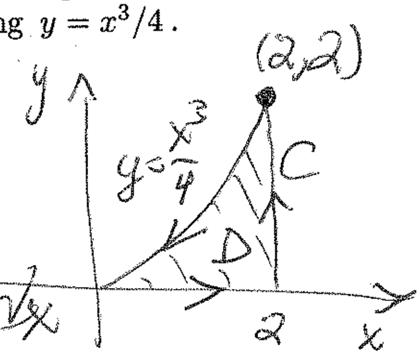
$$z = \rho\cos\varphi = 3\cos\tfrac{\pi}{6} = 3\tfrac{\sqrt{3}}{2}$$

$$r = \rho\sin\varphi = 3\cos\tfrac{\pi}{6} = 3/2$$



2. Use Green's Theorem to rewrite $\oint_C y\,dx + x^2\,dy$ as a completely set up double integral, where C is the curve formed by running from the origin along the x-axis to $x = 2$, then straight up, and finally back to the origin along $y = x^3/4$. Do NOT compute.

$$\iint_D (2x - 1)\,dA = \int_0^2 \int_0^{\frac{x^3}{4}} (2x-1)\,dy\,dx$$



$y = M$ $\quad \frac{\partial M}{\partial y} = 1$

$x^2 = N$ $\quad \frac{\partial N}{\partial x} = 2x$

(over →)

$M \qquad N$

3. Let $\mathbf{G} = \langle \frac{2xy}{1+x^2}, \ln(1+x^2) + \sin y \rangle$:

a. On what domain is $\mathbf{G}$ defined? Explain why $\mathbf{G}$ is a conservative vector field.

All x and y; $\mathbb{R}^2$ is simply conn.

$$\frac{\partial M}{\partial y} = \frac{2x}{1+x^2} \qquad \frac{\partial N}{\partial x} = \frac{2x}{1+x^2} + 0$$

equal, so $\vec{G}$ is conservative

b. Compute a potential function $\varphi(x, y)$ for $\mathbf{G}$.

$$\varphi = \int \frac{2xy}{1+x^2}\,dx = y\ln(1+x^2) + A(y)$$

in common

$$\varphi = \int (\ln(1+x^2) + \sin y)\,dy$$

$$= y\ln(1+x^2) - \cos y + B(x)$$

Compare $\varphi(x,y) = y\ln(1+x^2) - \cos y + C$

plain constant

c. Compute $\int_C \mathbf{G} \cdot d\mathbf{r}$, where C is any smooth path (*e.g.*, the line segment) from $P(0, \pi/3)$ to $Q(1, \pi)$. Why does the choice of path not matter?

Since $\vec{G}$ is ~~defined~~ known to be conservative, the choice of path does not matter.

$$\int_C \vec{G}\cdot d\vec{r} = \varphi(Q) - \varphi(P)$$

$$= (\pi \ln 2 - (-1)) - \left(\frac{\pi}{3}\overset{0}{\overset{\shortparallel}{\ln 1}} - \cos\frac{\pi}{3}\right)$$

$$= \pi \ln 2 + 1 + \tfrac{1}{2}$$