

For full credit you must show sufficient work that the method of obtaining your answer is clear. There is no need to "simplify" answers.

1. Let  $z = f(x, y) = 2 + xy + 3x^2$ ,  $\mathbf{a} = \langle 3, -4 \rangle$ , and P be the point  $(-1, 3)$ .

a. Compute the gradient of  $f$ .

$$\vec{\nabla} f = \langle f_x, f_y \rangle = \langle y + 6x, x \rangle$$

For (b),  $\vec{\nabla} f(P) = \langle -3, -1 \rangle$ .

b. Compute the directional derivative of  $f$  at P in the direction of  $\mathbf{a}$ .

Need a unit vector  $\hat{u} = \frac{1}{\|\vec{a}\|} \vec{a} = \frac{1}{\sqrt{9+16}} \vec{a}$

$$D_{\hat{u}} f(-1, 3) = \langle \frac{3}{5}, -\frac{4}{5} \rangle$$

$$\begin{aligned} \vec{\nabla} f(-1, 3) \cdot \hat{u} &= \langle -3, -1 \rangle \cdot \langle \frac{3}{5}, -\frac{4}{5} \rangle \\ &= -\frac{9}{5} + \frac{4}{5} = -1 \end{aligned}$$

c. What is the maximum value among all directional derivatives of  $f$  at P?

$$\|\vec{\nabla} f(P)\| = \sqrt{9+1} = \sqrt{10}$$

d. Give an equation for the tangent plane to the surface  $z = f(x, y)$  at the point  $(-1, 3, f(-1, 3))$ .

$x_0 = -1, y_0 = 3, z_0 = f(-1, 3) = 2 - 3 + 3 = 2$

Use  $\vec{N} = \langle f_x(-1, 3), f_y(-1, 3), -1 \rangle = \langle -3, -1, -1 \rangle$

$$-3(x+1) - 1(y-3) - 1(z-2) = 0$$

$$-3x - y - z = -2$$

OR  $z = L(x, y) = f(-1, 3) + f_x(-1, 3) \overset{(x+1)}{\text{over } x} + f_y(-1, 3) \overset{(y-3)}{\text{over } y}$

$$\begin{aligned} z &= 2 - 3(x+1) - 1(y-3) \\ &= 2 - 3x - y \end{aligned}$$

2. Consider  $w = g(x, y, z) = x^2z - yz^3$ .

a. The point  $Q(1, -1, 2)$  is on which level surface for  $w$  (or  $g$ )?

$$w = 2 - (-8) = 10$$

b. Determine an equation for the tangent plane to this surface at  $Q$ .

Use  $\vec{N} = \vec{\nabla}w = \vec{\nabla}g$  at  $Q$ .

$$\vec{\nabla}g = \langle g_x, g_y, g_z \rangle \quad \nwarrow \perp \text{ level surface}$$

$$= \langle 2xz, -z^3, x^2 - 3yz^2 \rangle$$

$$\vec{\nabla}g(Q) = \langle 4, -8, 1 - (-3)(4) \rangle$$

$$= \langle 4, -8, 13 \rangle$$

$$4(x-1) - 8(y+1) + 13(z-2) = 0$$

$$4x - 8y + 13z = 4 + 8 + 26 = 38$$