MATH 241 Spring, 2010 Quiz #5 Name: M^2

For full credit you must show sufficient work that the method of obtaining your answer is clear. There is no need to "simplify" answers.

1. The function $z = f(x,y) = \tan^{-1}(xy)$ is defined everywhere on the (x,y)-plane except along the x and y-axes.

   a. Compute $\operatorname{grad} f = \vec{\nabla} f$. $\vec{\nabla} f = \langle f_x, f_y \rangle$

$$= \left\langle \frac{1}{1+x^2y^2}\cdot y \,,\; \frac{1}{1+x^2y^2}\cdot x \right\rangle$$

   b. Compute $\dfrac{\partial^2 z}{\partial x \partial y} = f_{yx}$; then give $\dfrac{\partial^2 z}{\partial y \partial x} = f_{xy}$.

$$f_{yx} = \frac{\partial}{\partial x}\left(\frac{x}{1+x^2y^2}\right) = \frac{(1+x^2y^2)(1) - x(2xy^2)}{(1+x^2y^2)^2}$$

$$\underbrace{f_{xy} = f_{yx}}_{\text{all you really need to say!}} = \frac{1+x^2y^2-2x^2y^2}{(1+x^2y^2)^2}$$

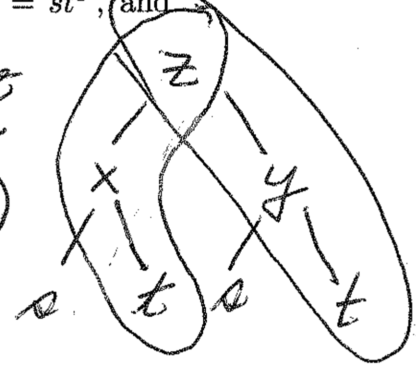
$$= \frac{1-x^2y^2}{(1+x^2y^2)^2}$$

2. Compute $\dfrac{\partial z}{\partial t}$ in terms of x, y, s, and t if $z = xy^2 \sin x$, $x = st^3$, and $y = s^4 t$.

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t} \quad \text{product rule}$$

$$= \left(xy^2\cos x + (1)(y^2 \sin x)\right)(3st^2)$$

$$+ (2xy\sin x)(s^4)$$



Multiply down the chains; then add up the contributions from each chain.