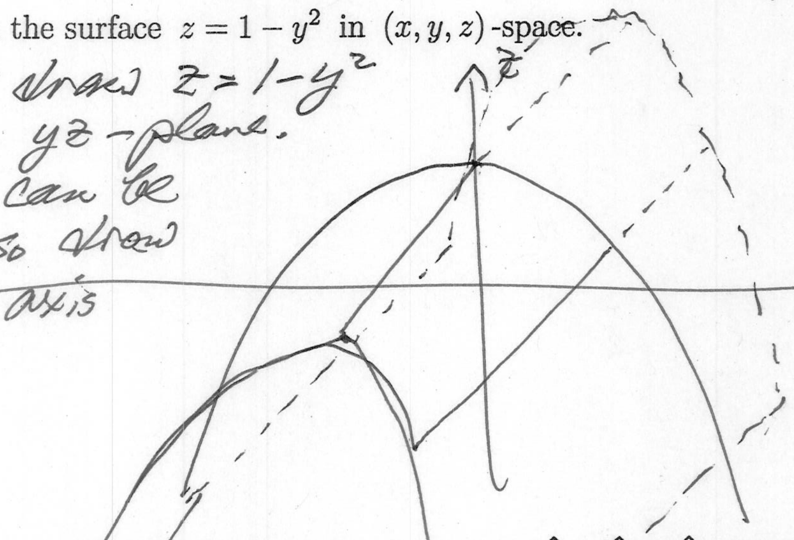
MATH 241 Spring, 2010 Quiz #1 Name: M^2

1. Sketch the surface $z = 1 - y^2$ in (x, y, z)-space.

First draw $z = 1 - y^2$ in the yz-plane. Then x can be anything so draw lines $\parallel$ x-axis

It's like an arch or a tunnel roof.



2. Find the terminal point Q of $\mathbf{v} = \langle 1, 2, -3\rangle = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$ if the initial point P is $(-2, 1, 4)$. Also find the length of $\mathbf{v}$.

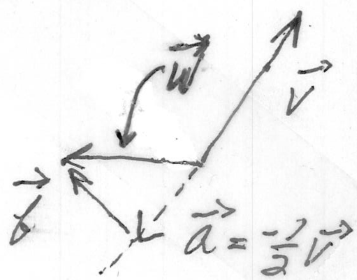
Q is the point $(-2+1, 1+2, 4-3)$
$= (-1, 3, 1)$

$\|\vec{v}\| = \sqrt{1^2 + 2^2 + (-3)^2} = \sqrt{14}$

3. Compute the orthogonal projection of $\mathbf{w} = \langle 1, -1, 2\rangle$ on $\mathbf{v} = \langle 1, 2, -3\rangle$ (that is, $\mathbf{a} = \mathrm{proj}_{\mathbf{v}}\,\mathbf{w}$, the component of $\mathbf{w}$ in the direction of $\mathbf{v}$). Then compute the component of $\mathbf{w}$ that is orthogonal to $\mathbf{v}$; call this vector $\mathbf{b}$. Finally, verify that $\mathbf{a}$ is orthogonal to $\mathbf{b}$.

$$\vec{a} = \left(\frac{\vec{w}\cdot\vec{v}}{\vec{v}\cdot\vec{v}}\right)\vec{v} = \frac{1-2-6}{14}\langle 1, +2, -3\rangle$$

$$= -\tfrac{1}{2}\langle 1, 2, -3\rangle = \langle -\tfrac{1}{2}, -1, \tfrac{3}{2}\rangle$$



$$\vec{b} = \vec{w} - \vec{a}$$

$$= \langle \tfrac{3}{2}, 0, \tfrac{1}{2}\rangle$$

$$\vec{a}\cdot\vec{b} = (-\tfrac{1}{2})(\tfrac{3}{2}) + 0 + \tfrac{3}{2}(\tfrac{1}{2})$$

$$= 0$$

so $\vec{a} \perp \vec{b}$