MATH 241 Spring, 2010 Exam #3 Name:____________________

For full credit you must show sufficient work that the method of obtaining your answer is clear. There are 100 points. Some reference formulas: $r = \rho \sin\phi$, $z = \rho\cos\phi$, $x = r\cos\theta$, $y = r\sin\theta$.

1. (15 points) Re-express the integral $\displaystyle\int_0^1 \int_{3y}^3 \cos(x^2)\,dx\,dy$ with the order of integration reversed, and compute the easier version.

2. (10 points) A vector field $\mathbf{F}$ is illustrated below, with a path C made up of C_1, C_2, C_3 and C_4 in this order. Determine if $\int_C \mathbf{F}\cdot d\mathbf{r}$ is negative, positive, or zero. Does $\mathbf{F}$ have the "independence of path" property in the region shown? Explain, briefly.

3. (15 points) SET UP completely (*i.e.*, give the limits of integration determined by D and an explicit form of dV, with everything in a consistent order), but do NOT compute, a triple integral for the volume of the region D bounded by the xy-plane, the plane $z = x + 2$, and inside the cylinder $x^2 + 4y^2 = 4$.

4. (10 points) SET UP, but do NOT compute, the triple integral in spherical coordinates that gives the volume inside the sphere $x^2 + y^2 + z^2 = 25$, and **outside** the double cone $z = \pm r$ $(i.e.,\ z^2 = x^2 + y^2)$.

5. (25 points) Compute $\oint_C \mathbf{G} \cdot d\mathbf{r}$ for $\mathbf{G} = \langle 3xy^2, -3x^2y \rangle$, where C is given by the circular arc of $x^2 + y^2 = 1$ clockwise from $(0,1)$ to $(1,0)$, then the segment from $(1,0)$ to $(2,0)$, then the circular arc of $x^2 + y^2 = 4$ counterclockwise from $(2,0)$ to $(0,2)$, and finally the segment from $(0,2)$ back to $(0,1)$. (Hint: use a major result and then use polar coordinates.)

6. (25 points) Let $\mathbf{F} = \langle ye^{xy}, xe^{xy} \rangle$ and C be the path given by $x = 1 - 2t$, $y = -1 - t$ for $0 \leq t \leq 1$.

a. What is the domain of $\mathbf{F}$? At what point P does C begin and at what point Q does it end?

b. Is the integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ is independent of path? Briefly explain.

c. Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$. This can be done by straightforward direct calculation, or if you can justify that there is a potential function $\varphi(x, y)$ for $\mathbf{F}$, then compute it and use it.