b. How can you be certain that (I) is NOT the contour diagram of a plane? (Hint: if $z = ax + by + c$, with a, b, and c constants, what are $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$?

3. (35 points) Suppose $w = h(x, y, z) = z^2 + \ln(1 + xy)$, $x = s^3 t$, $y = s^2 \sin(st)$, and $z = 4 - t^2$.

a. Compute $\text{grad}\, h = \vec{\nabla} h$ in terms of x, y and z.

b. Compute the directional derivative $D_{\hat{\mathbf{u}}} h(P)$ at the point $P(1, 3, -1/2)$ in the direction of $\mathbf{a} = \langle 2, 2, -1 \rangle$.

c. Give the maximum value for any directional derivative of h at P.

d. Which level surface for w (or h) is the point $Q(1, 1, -1)$ on? Give an equation for the tangent plane to this level surface at Q.

Level surf: $w = (-1)^2 + \ln(1+1) = 1 + \ln 2$

Obviously for TP normal at Q you need to use the coordinates of Q, not P!

$\vec{N} = \vec{\nabla} h(Q) = \langle \frac{1}{2}, \frac{1}{2}, -2 \rangle$

Eqn for TP is $\frac{1}{2}(x-1) + \frac{1}{2}(y-1) - 2(z+1) = 0$

e. Compute $\dfrac{\partial w}{\partial s}$ in terms of x, y, z, s, and t, using the multivariable chain rule.

or $\frac{1}{2}x + \frac{1}{2}y - 2z = 3$