

Mathematical Modeling for the Life Sciences

Instructor. Matt Miller, office: LeConte 300I, hours: MW 1:30-3:00, and by appointment, phone: 777-4224 (leave message), e-mail: miller@math.sc.edu, web: <http://www.math.sc.edu/~miller/172>. Notice that these are ordinary email and website addresses, NOT accessed through Blackboard.

Text. Required: *A Primer of Ecology* by Nicholas Gotelli, Sinauer Press, 2007, 4th ed., 2008. As you can tell from the title this text will support BIOL 301, if taken concurrently, and will be a great warm-up if taken before 301. That said, I will try to work in some models from other parts of biology, although, truth to tell, it is a lot easier to understand population counts than concentrations or moles (the chemical kind), etc. This text contains some genuine biology, but you don't need any specialized knowledge of ecology to use it. You will also need a graphing calculator (TI-83 preferred) that can handle sequences (discrete models, recurrence relations, discrete dynamical systems, these all mean much the same thing).

Prerequisite. You must have earned a grade of C or better in MATH 122, 141, or an equivalent course to be approved by the instructor. This pre-requisite is essential.

Overview. Unless you have taken MATH 122 here at USC-Columbia you will possibly find that this course is very different from other math courses that you have taken. The course is not intended to be merely a collection of mathematical tools that you might someday, maybe, use in biology; rather it is an introduction to a way of thinking about biology that uses mathematics to create a conceptual and unified approach. Model building and model analysis, both quantitative and qualitative, will be as important as model solving. We form a mathematical model of a changing real world situation, such as population growth, use a variety of methods to analyze it, and then interpret our calculated results in the context of the original problem. We will analyze problems by using a blend of verbal, pictorial, numerical, graphical, and analytic methods (manipulation of formulas). Modeling is more comprehensive than problem solving; we will learn how models are built, as well as how to "read" them. One of the goals of the course is to enable you to look at scary formulas and to be able to dissect them into small and comprehensible pieces, so that even if you don't "do the math" yourself, you will be able to follow what has been done. Finally, in the real world, solutions must be communicated effectively, both in writing and orally, and you will get some practice doing this.

Learning Outcomes. Students will understand the concepts of and be able to solve problems drawn from biological modeling with differential and difference equations; techniques of model modifications; analytic, numerical, and graphical solution methods; equilibria, stability, and long-term system behavior; geometric series; vectors, matrices, eigenvalues, and eigenvectors, with applications principally to population dynamics and compartment models.

Course content. We begin with an introduction to the idea of modeling a system (why do it?), how to build models starting with really simple ones and gradually building up to

more complex ones, and how to interpret models that you can pull off the shelf. Models come in two flavors: discrete and continuous; we will explore how these are similar, how they are different, and why one might choose one over the other. The discrete ones are usually called difference equations; the continuous ones are called differential equations (DE's) and these you have already been solving in your calculus classes! Every time that you concluded something about a function, say $P(t)$ population as a function of time, from information about its rate of change $P'(t)$ you were actually solving a DE. One of our main tools will be the study of the equilibria (stationary values) of a model, what these are, how to compute them, how to determine if they are stable or not, and how they are related to the long term behavior of a system. Put crudely, we want to use models make predictions such as whether a population (or an epidemic) will boom (turn into a pandemic), go extinct (die out), or fluctuate around a certain level. Next we will learn a little bit about matrices and vectors, emphasizing the intuitive geometric point of view, and then go on to selected topics involving multi-species or metapopulation models that use these basic tools. We will be very selective in what we cover; although the book is small, it is packed and we won't cover more than the first six chapters, perhaps with chapter 8 in place of chapter 4. Notice that the organization is by biological topic, not mathematical. All along we will work with tables of data, or verbal descriptions of problems, to build our models, and we will use our calculators to make educated guesses about the **qualitative behavior** of the solutions. You will be expected to **gradually recognize for yourself** when the use of technology is appropriate, and when hand computation and exact algebra or calculus is called for.

Grades. Three major tests will be given, each worth 100 points, on Thursday, 15 September (day 9); Tuesday, 18 October (day 18); and Tuesday, 22 November (day 27). The final exam is scheduled for Wednesday, 7 December, at 2 pm and is optional. It will be divided into three parts corresponding to the three exams, and you may do as many of these parts as you like. At least seven eight-point quizzes will be given; the six highest scores will be counted. **No make-ups will be given for quizzes or exams**, but each of the three segments of the final exam (scaled up to be out of 100) will be used to replace the score for the corresponding in-class exam, provided this helps you. There will be two group projects, each worth 26 points. The first will be due on 27 Sept. (day 12); the second on 10 November (day 24). "Board points" will count as bonus points. The homework total will be scaled to 25 points. A total of 425 points may be earned:

Exams	300	(three at 100 pts each)
Quizzes	48	(best six)
Projects	52	(two at 26 pts each)
Homework	25	(scaled score)

Letter grades will be announced separately for each exam, and for the overall quiz and project totals. They will generally fall close to the scale 85–100 A, 75–84 B, 65–74 C, 55–64 D, below 55 F on a percentage basis, but will vary up or down. Note that the deadline to drop this course without a grade of WF is Thursday, 13 October; you should have a pretty good idea before then how you are doing.

Collaboration. One of the goals of this course is to learn how to communicate mathematical ideas. You will be expected to work with one another in class and I expect that you will do so on homework and projects. However, you will have to take the exams individually, so don't get too dependent upon one another. Homeworks must be written up on your own, even if the work was done in collaboration. According to the USC Student Handbook code of student academic responsibility, "**the first law of academic life is intellectual honesty.**" I expect this of all of you. If you are ever in the least bit uncertain about the ground-rules, ask for clarification!

Attendance. Regular attendance is crucial for success in this course. Ten bonus points will be awarded for perfect attendance and 5 for only one absence. No excuses will be considered in this regard. This class has 29 meetings; university policy states that if more than 10% of the meetings are missed, whether excused or unexcused, then the instructor may impose a penalty. I intend that this be a very rich and varied class, often with non-lecture activities. If you feel that a class is nothing more than a series of exams and some assignments to be turned in, with attendance optional, then this is not the class for you. If you miss 4 or more class sessions, I will lower your grade by half a grade point (from an A to a B+, or a C+ to a C, for example), and if you miss 6 or more classes (that's three weeks!), your grade will drop by a full grade point. If you do miss a class, you can find homework and a very brief synopsis on the class home page <http://www.math.sc.edu/~miller/172>. Be aware that I often take attendance silently: if you don't turn in a quiz, or I pass back a quiz or exam and you do not pick it up, I will assume you were not in class; so if you come in late you should always check to see if I have marked you absent.

Having fun doing math. This might be a new concept for you, but I hope that the model building and analysis, the various verbal, numerical and graphical approaches, and the general de-emphasis on rote problem solving and formula memorization (there is some, but not much), will be stimulating. In many cases there will be more "thinking about the math" than "doing the math." This might be a little unsettling at first, because it means that you won't be doing the same problem over and over again, but will be called upon to think most problems through from scratch. I think this material is really beautiful, and I hope you will too.