**Math 172** **Worksheet 6** **Fall, 2011**

## Geometric series, more graphical analysis

1. Compute the sum of the series or state that no sum exists; and explain why or why not. In most cases (especially b., but also c., d.) it is a good idea to write the series in the $\cdots + \cdots$ form to see better how to choose the $a$ and the $r$ for the geometric series formula $\sum_{n=0}^{\infty} ar^n =$ either $a/(1-r)$ (when does this happen?) or no sum exists (when does this happen?). Alternatively in some cases (say for example c., d.) it is handy to find the sum of the whole series, assuming that the summation index started at 0, and then subtracting off the missing pieces.

   a. $\sum_{n=0}^{\infty}(5/2)(-2/3)^n$

   b. $\sum_{k=0}^{\infty}(-2/3)(5/2)^{k+1})$

   c. $\sum_{k=1}^{\infty}(3/4)(5/3)^k$

   d. $\sum_{j=2}^{\infty}(7/4)(-3/4)^j$

2. A reproductive adult in the oldest stage produces 15 offspring a year, and has a survival rate of 70%. What is the expected lifetime production of of an individual beginining as she enters this stage. Same question for a reproductive adult in the oldest stage who produces 7 offspring a year, and has a survival rate of 86%. Which factor is more significant in the long run: annual fertility or annual survivorship? How does the calculation demonstrate this?

3. In a predator-prey continuous model system, where humans are the predators, we generally say that we have a "harvesting" model. A victim population $V$ that grows logistically with intrinsic rate $r$ and carrying capacity $K$ suffers loss to a harvester whose functional response curve is constant, say $H$. Give the formula for $dV/dt$. Superimpose the loss rate curves on top of the logistic growth rate curve for low, medium and high values of harvesting rate $H$, and interpret the equilibria that you find in biological terms and in stability terms. You might want to make separate diagrams for each case for clarity, instead of using different colors. Is it possible for there to be no equilibrium? How would you interpret this? Suppose the opposite is happening: for boosting an endangered species we humans introduce a new individuals at a constant rate. What is the predicted effect on the equilibria? (Hint: think of this as negative harvesting!)

4. A chemical reaction involves constituents A, B and C. Each minute 15% of A is converted into B, while 25% of B converts naturally back into A. Meanwhile 15% of B converts into C.
   a. Write a system of discrete model equations and put these also into calculator form for the changing amounts of A, B, and C.
   b. To keep the system running 3 units of B are added each minute Write revised model equations for the changing amounts of A, B, and C. Without doing any calculations, explain why the amount of C must be steadily increasing if there is any A in the system at all.

5. Here is a table of values for a 2-variable dynamical system

| $n$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $u_n$ | 3 | 1 | 2 | 3 | 6 | 3 | 1 | 2 | 3 | 6 | 3 |
| $v_n$ | 1 | 4 | 7 | 10 | 6 | 1 | 4 | 7 | 10 | 6 | 1 |

   Plot $u_n$ and $v_n$ against one another on one graph, and label the points with the values of $n$ from 0 to 10. Plot $u_n$ and $v_n$ on a single graph against $n$ from 0 to 10. If you were told that this system has an equilibrium point $(4, 6)$, based on the available evidence, how would you classify this equilibrium?

6. Here is a table of values for a 2-variable dynamical system

| $n$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| $u_n$ | 3 | 1 | 5 | 6 | 3 | 1 | 5 | 6 | 3 |
| $v_n$ | 4 | 8 | 9 | 3 | 4 | 8 | 9 | 3 | 4 |

   Plot $u_n$ and $v_n$ against one another on one graph, and label the points with the values of $n$ from 0 to 9. Plot $u_n$ and $v_n$ on a single graph against $n$ from 0 to 8. What conclusion do you draw about this dynamical system, at least with the initial condition $u_0 = 3$ and $v_0 = 4$?

7. A victim population has a growth rate curve (light line) as shown (supplemental sheet).
   a. At low population levels the growth rate is not logistic; how is it different?
   b. Superimposed on this graph is a heavy curve indicating the loss rate due to moderate predation by a predator that exhibits a type II functional response. Label the equilibrium values for $V^*$ on the graph, determine if each is stable or unstable, and indicate verbally or by arrows how $V$ will change if it falls just slightly off each equilibrium value.

8. In a predator-prey continuous model system, the prey ("rabbits") population $R(t)$, measured in hundreds, grows with a per capita rate of 0.16 yr $^{-1}$ in the absence of predators ("foxes"). The net growth rate is reduced by predation: 80% of the rabbit-fox interactions result in the loss of a rabbit (model this with a mass action interaction with a coefficient of 0.8 and an appropriate plus or minus sign). The fox population $F(t)$, measured in tens, declines at a per capita rate of 0.25 yr $^{-1}$ in the absence of rabbits. The net growth rate, however, is increased by predation: each fox-rabbit interaction increases the fox population by 0.1 (in other words it takes consumption of 10 rabbits to produce one fox).
   a. Write the model equations for this system.
   b. One equilibrium is of course $R = 0 = F$. Find another one.

9. The discrete model system given by $\begin{cases} u_n = 2u_{n-1} - 5v_{n-1} + 20 \\ v_n = u_{n-1} - 2v_{n-1} + 10 \end{cases}$ has an equilibrium of $(5,5)$. Determine the behavior of this system starting from $n = 0$ and going until you see a pattern (you need to pick values for $u_0$ and $v_0$), and describe this pattern. You don't need to write down all the values that you compute, but a selection from your table, or a graph should be provided as evidence.

10. We have a predator-prey (fox-rabbit) discrete model in which

$$R_{n+1} = 1.3R_n - 0.2F_n$$
$$F_{n+1} = 0.15R_n + 0.9F_n$$

Give a formula for the fox population $F_k$ in terms of $k$ and the initial population $F_0$ if there are no rabbits. What happens to the fox population $F_k$ in the long run? Do the same thing for the rabbits, under the assumption that there are no foxes. Does this model have an equilibrium, and if so, what is it? Use your calculator to find evidence for stability or instability of the system.