WS 3 #6

$M' = 0.03\,M - 60$ tons/yr
$M(0) = 500$ tons } given
Seeking $M(20)$ tons at $t = 20$ yr.

Equil: $M' = 0$ $\quad M^* = \frac{60}{.03} = 2000$

Note that $M'(0) = (0.03)(500) - 60$
$= -45 < 0$
so $M(t)$ starts out decreasing (a good sign that the removal is working)

Now $M(t+\Delta t) = M(t) + (\Delta t)\,M'$. Let's take 100 steps so $\Delta t = \frac{20}{100} = 0.20$.

This is 5 steps per year

We have in the calculator:
$u(n) = u(n-1) + 0.2 * (0.03 * u(n-1) - 60)$

Then we have

	year	n	u(n)	
	0	0	500	
Clearly $M(t) \to 0$,	1	5	454.5	
but it cannot	7	35	150.6	
go negative!	9	45	36.6	
Some value in the	9.6	48	1.1	Must actually be 0.
vicinity of 9.7 yr,	9.6^+	48^+	0	
M is wiped out.	9.8	49	−10.9	

WS 3 #7 $Q_{t+1} = 0.4Q_t - 48$

Equil. when $\Delta Q = 0$ or when $Q_t = Q_{t+1} = Q^*$ is unchanging.

$$Q^* = 0.4Q^* - 48, \quad Q^* = -\frac{48}{0.6} = -80$$

In the calculator $u(n) = 0.4 * u(n-1) - 48$
First we use $u(nMin) = 60$.
~~[illegible]~~
It appears that $u(n) \to -80$, suggesting that $Q^* = -80$ is stable. Next we use $u(nMin) = 180$. It again appears that $u(n) \to -80$.

Bonus: if you checked below -80, great! For instance if $u(nMin) = -120$, then once again $u(n) \to -80$, so stability is really confirmed.

Note that there are no approximations or Δt's involved in a discrete model, we can go directly to the calculator. But as we shall see there are other twists & turns with discrete models (try replacing 0.4 with -0.4 or -1.4 and re-doing the problem)!