Math 172 WS 3 **Fall, 2011**

Solving model equations approximately and graphically

1. Sketch all the graphs of $P(t)$ if $\frac{dP}{dt} = rP(1 - \frac{P}{70})$. Label your graphs **clearly** with a, b, c, d.
   a. if $r = 0.4$ and $P(0) = 40$
   b. if $r = 0.4$ and $P(0) = 10$
   c. if $r = 0.2$ and $P(0) = 90$
   d. if $r = 0.2$ and $P(0) = 10$
   e. What is this kind of model called? What is the biological significance of the number 70? of the value of r?

2. Squid are commercially important for human consumption, especially around the Mediterranean and in SE Asia; they also play a large role in many marine food chains. Suppose that a local squid population, measured in tons, is controlled by the continuous model equation $S'(t) = 0.0007S(100 - S)(S - 10)$ tons/year. It is easy to separate the variables of this model equation, but hard to do the integration.
   a. Compute the equilibrium values for S.
   b. From here on, assume that the initial population is $S(0) = 30$ tons. Estimate the squid population in 5 years, using Euler's method with 5 steps (by hand, if you must).
   c. Estimate the squid population in 5 years, using 60 steps (this amounts to getting a month by month estimate; what is the new value of Δt?). Hint: To get this into your calculator remember to set the MODE on Seq, and use Y= to give the formula for $u(n)$ interms of $u(n-1)$. Set $n\text{Min} = 0$, $u(n\text{Min}) = 30$, and remember that $\Delta S \approx (\Delta t)S'(t)$. Now ΔS becomes $\Delta u = u(n) - u(n-1)$, and S or $S(t)$ becomes $u(n-1)$, which is used in the formula for $S'(t)$. Make sure that Tblset is OK, then use Table (and Graph, if you like).
   d. Estimate the squid population in 5 years, using 100 steps (give your calculator time to compute!).
   e. Repeat using other initial values as necessary to determine if each of the equilibrium values is stable or unstable.

3. For a continuous model, how can you tell if Δt is small enough (the number of steps is large enough) to give a moderately accurate prediction?

4. A fish population $F(t)$, measured in thousands, is controlled by the continuous model $\frac{dF}{dt} = 0.0005F(120 - F)(F - 20)$ thousand fish/year. It is easy to separate the variables of this model equation, but hard to do the integration.
   a. Find the equilibrium values of F, and examine the long term behavior of $F(t)$ subject to various initial conditions for $F(0)$. Which are stable,

which are unstable? How would you describe the various values from a biological point of view?

b. Assume now that the initial population is 40 thousand fish. Apply stepwise estimation of $F(t)$ using Euler's method to estimate the population after 3 years. Since we are measuring in thousands, it is reasonable that you may get decimal answers; round off to **three** decimal places. Use 30 steps (so $\Delta t =$ ______) to approximate $F(3)$.

c. Now suppose the initial population is 10 thousand fish. Use 30 steps to approximate $F(3)$.

5. Due to poor environmental conditions, brook trout are declining continuously by 4% a year. A particular stream is stocked with 5 thousand farm-raised trout a year. Write a model equation to describe the net rate of change of the brook trout population over time. Assume also that in our baseline year the population is estimated to be 200 thousand trout. Solve this model equation approximately by using your calculator, and discuss the long term implications for sport fishing in this stream. Give details such as the total number of years you consider, the number of steps and the value of Δt, and the model equation that you enter into the calculator. Could they get away with not artificially stocking the stream? What if the initial population is only 100 thousand trout?

6. An invasive water weed is growing continuously in a local lake at an intrinsic rate of 3% a year. Lake management dredges up 60 tons of the weed each year. If $M(t)$ is the mass of the weed measured in tons, and t is measured in years, write the model equation for $M'(t)$. Determine if there is an equilibrium, and if so, compute it. Compute an approximation the explicit solution for 20 years from now if the invasion was first detected when the weed mass now is estimated to be 500 tons. Then use appropriate time scales to determine if the weed grows out of control, if it approaches the equilibrium value, or if the harvesting eventually brings about elimination of the weed. In the first case, determine when $M(t) = 1500$; in the third case determine when the weed is completely eliminated.

Discrete models are even easier to put into your calculator. Just remember to replace t (or n) by $n-1$ and $t+1$ (or $n+1$) by n, and the variable P, Q, whatever, by u, v or w. (You don't have to keep re-using u; if you want you can leave it but "de-activate" it by turning off the highlighted = sign.) Equilibrium values are found when, say, $\Delta Q = 0$ if the model is given in difference form, or when $Q_{t+1} = Q_t$ if the model is given in updating form. If the dependent variable is Q, then the equilibrium value(s) is/are denoted by Q^*.

7. A quantity Q_t is governed by the discrete model equation $Q_{t+1} = 0.4Q_t - 48$. Find the equilibrium value Q^* (this is just doing the math, so don't worry if it seems unrealistic biologically). Use initial values of $Q_0 = 60$ and $Q_0 = 180$ to determine long term behavior of the system, and stabiltiy or instability of the equilibrium.

8. We are given a discrete model $P_{n+1} = (-0.9)P_n + 95$ with $P_0 = 70$. What happens to P_n as $n \to \infty$? Does it increase, decrease, oscillate, tend towards or away from the equilibrium? Conclude whether the equilibrium is stable or not.

9. We are given a discrete model $S_{n+1} = (-1.3)S_n + 161$ with $S_0 = 100$. What happens to S_n as $n \to \infty$? Does it increase, decrease, oscillate, tend towards or away from the equilibrium? Conclude whether the equilibrium is stable or not.

10. How are graphical solutions of discrete models different from those of continuous models?