Math 172 **Worksheet 2** **Fall, 2011**

Setting up and manipulating model formulas

1. In 2009 Jordan had a population of 6.053 million people. The per capita growth rate was 24.1 people per 1000 people per year. Write a continuous model equation for the net growth rate of Jordan. Give the explicit solution and use your result to predict the population in 2011 and in 2019 (assuming the per capita growth rate does not change). How long will it take for the population to double? The actual population in 2011 is 6.199 million; how might you account for the discrepancy?

2. Suppose a population $B(t)$ of bacteria is growing over time so that the **per capita** rate of increase is 0.008/day. Assume that bacteria reproduce continuously.
   a. Write the model equation that describes this situation.
   b. Write the explicit solution for $B(t)$, which is an equation, for this model equation.
   c. An initial population of 30 mg (we weigh bacteria, we don't count them!) grows to what size in 99 days?

3. A 50 mg initial dose of quinine is given intravenously to a patient. Quinine is continuously metabolized and leaves the bloodstream at a rate of 6% an hour.
   a. Write the model equation for this process.
   b. Find the explicit solution equation, and determine how much quinine remains in the bloodstream after 12 hours.
   c. At what time does just 5 mg of quinine remain?

4. Do problem 1.1 in Gotelli.

5. Do problem 1.2 in Gotelli in two ways: with a discrete model, with a continuous model. Don't fudge it as in the answer given in the text by mixing the two approaches together. Which do you think reflects biological reality better?

6. Do problem 1.3 in Gotelli. What is the point of this problem? Think about what the fact that ln of population grows linearly tells you about how the population itself is growing (that might not have been immediately apparent from a curved graph). What about estimation of r: how is this easy using the line, but otherwise hard?

7. Do problem 1.4 in Gotelli, again assuming first a discrete growth model and then a continuous one. Which do you think reflects biological reality better?

8. The **net** rate of change of a continuously growing fruit fly population is directly proportional to the population $F(t)$ itself. At the beginning of the experiment when there are 1000 flies, we observe that this rate is 14 flies/week.
   a. Determine the value of the constant of proportionality r. In this case the **per capita** rate of change (with units) has the value _______ / _______ . Give the continuous model equation using the variables F, t, and explicit constants.
   b. Your lab partner claims that the explicit solution equation for this population is $F(t) = 1000e^{0.28t}$. If so, then $F(0) =$ _______ and the model equation is the one you gave in part a. Is your partner's claim fully correct, partially correct, or entirely wrong, and how can you demonstrate this?

9. Due to poisoning a mouse population is declining continuously at a rate of 5% a month. But food supplies are ample, so as the population declines migrants move in steadily at a rate of 8 per month.
   a. Write a continuous model equation to describe this process.
   b. Is there a steady state population (equilibrium) in which death of the mice is exactly balanced by growth of the population from immigration? If so, compute this level. What happens if the population starts below this level? What happens if the population starts above this level? Suggestion: while you cannot get an explicit solution for now, you can use Euler's method to estimate solutions over a period of several months.

10. A population F_t of fruitflies (Drosophila) depends on time t . The initial population is $F_0 = 1000$ flies. The population is censused once every two (2) weeks. Over this period the natural rate of increase is 0.8%. At each census 40 flies are removed from the population and sacrificed for genetic analysis. Write the discrete model equation for this process. Is there a steady state population (equilibrium) in which removal of flies is exactly balanced by growth of the population? If so, compute this level. What happens if the population starts below this level? What happens if the population starts above this level?

11. A population $Q(t)$ is given by $Q(t) = 400e^{-0.006t}$.
   a. What is the value of Q_0? What is the underlying continuous model equation?
   b. What can you say about $Q(t)$ as time runs on and on?
   c. Describe the graph of $\ln(Q)$ as a function of t.
   d. Suppose instead that $Q(t) = 900 - 400e^{-0.006t}$. Work parts (a) and (b) over again.

12. A population $A(t)$ is growing continuously at $r = 2\,\%$ per year, and simultaneously there is a continual annual harvest of $h = 12$ million a year.

a. Write the model equation for this process. Note we do not have an explicit solution to work with.

b. Find the equilibrium value for A. Predict what will happen if the population starts with $A(0) = 200$ million. Predict what will happen if the population starts at $A(0) = 300$ million.

c. Get more precise predictions in each case over a 5 year period by using Euler's method with 20 steps. In this case $\Delta t =$ _______ years. Keep reading before plunging in!

d. You don't really want to do all those steps by hand, so let's see if we can adapt this **discrete approximation** to the continuous model on the TI calculator. We will let n count steps, as on the calculator. If we replace $A(t + \Delta t))$, i.e., the updated value, by $u(n)$, then we should replace $A(t)$ by $u($ _______ $)$. We will replace ΔA by Δu, and we have $\Delta A \approx A'(t)(\Delta t) = \frac{dA}{dt}(\Delta t)$. Now we can rewrite $A'(t)$ in terms of $A(t)$ and some numbers, and we know Δt, so we can write $A(t + \Delta t) = A(t) + \Delta A$ out in terms of $A(t)$ and some numbers. From this we can write an updating equation for U to put in the calculator. Suggestion: write $(\Delta t)A'(t)$ instead of $A'(t)(\Delta t)$ to make sure you get all the parentheses right.

e. Now predict 20 years ahead in each case ($A(0) = 200$, $A(0) = 300$) using the same Δt, except now you need _______ steps. Alternatively, predict 20 years ahead still using 20 steps, but now $\Delta t =$ _______ has to be modified in your formula. Which calculation is more accurate?

Here is the outline of a table to help you get started.

n	t	$A(t)$	$A'(t)$	$\Delta A \approx A'(t)\Delta t$	$A(t+\Delta t)$
0	0	200	-8	-2	198
1	0.25	198	-8.04	-2.01	195.99
2	0.5	195.99			
3	0.75				
20	5				