

Recall that the geometric series  $\sum_{n=0}^{\infty} ar^n$  has a sum  $S_{\infty} = a/(1-r)$  under a certain condition on  $r$ , which you should verify, and fails to exist otherwise.

1. Compute the equilibrium point  $(u^*, v^*)$  of the two-variable discrete model

$$u_n = 3u_{n-1} - 2v_{n-1} - 4$$

$$v_n = 5u_{n-1} - 3v_{n-1} - 28$$

$$\begin{cases} u^* = 3u^* - 2v^* - 4 \\ v^* = 5u^* - 3v^* - 28 \end{cases}$$

$$\begin{cases} 4 = 2u^* - 2v^* \\ 28 = 5u^* - 4v^* \end{cases}$$

$$\begin{cases} -8 = -4u^* + 4v^* \\ 28 = 5u^* - 4v^* \end{cases}$$

$$20 = u^*$$

$$(u^*, v^*) = (18, 20)$$

2. A patient is given a 60 mg dose of a drug at regular intervals. In the time in between the drug declines to 15% of the amount present.

- a. At the time of the third dose (two time intervals) how much of the second dose remains?  $60(0.15) = 9$

- b. How much of the original dose remains?  $60(0.15)^2 = 1.35$

- c. Including the third dose, how much drug is in the bloodstream?  $60 + 60(0.15) + 60(0.15)^2 = 70.35$

- d. What is the long term amount of drug in the bloodstream assuming that the dose continues to be repeated?

$$\sum_{n=0}^{\infty} 60(0.15)^n \quad |0.15| < 1$$

$$= \frac{60}{1-0.15} = \frac{60}{0.85} = 70.59$$

- e. If 70 mg of the drug is needed to be effective, but above 75 mg is fatal, is this dosing pattern effective and safe?

At a long term level of 70.59 (above 70) it is effective and (below 75) it is safe.