

For full credit you must show sufficient work to justify your answer.

1. Compute the equilibrium point (u^*, v^*) of the discrete model system

Set $u_n^* = u_{n-1}$ $u_n = 3u_{n-1} - 2v_{n-1} - 4$
 $v_n^* = v_{n-1}$ $v_n = 5u_{n-1} - 3v_{n-1} - 28$

$$\begin{cases} u^* = 3u^* - 2v^* - 4 \\ v^* = 5u^* - 3v^* - 28 \end{cases}$$

$$\begin{aligned} (-2) \times & \begin{cases} 4 = 2u^* - 2v^* \\ 28 = 5u^* - 4v^* \end{cases} \\ & \begin{cases} -8 = -4u^* + 4v^* \\ 28 = 5u^* - 4v^* \end{cases} \end{aligned}$$

Add up: $20 = u^*$
 Then $2v^* = 2u^* - 4$
 $v^* = u^* - 2 = 18$
 $(u^*, v^*) = (20, 18)$

2. Consider the following continuous model of a predator-prey system.

$$\begin{aligned} \frac{dV}{dt} &= 0.6V \left(1 - \frac{V}{100}\right) - 0.02VP = [0.6 \left(1 - \frac{V}{100}\right) - 0.02P]V \\ \frac{dP}{dt} &= -0.4P + 0.005VP = (-0.4 + 0.005V)P \end{aligned}$$

Answer:
 $(V^*, P^*) = (80, 6)$

- What kind of growth does the victim population exhibit if there are no predators (i.e., $P = 0$)? What kind of long term trend is there for the predator if there are no victims (i.e., $V = 0$)?
- Compute the equilibrium (V^*, P^*) other than $(0, 0)$ for the predator-prey system. Suggestion: find V^* first.

(a) If $P=0$, then $\frac{dV}{dt} = 0.6V(1 - \frac{V}{100})$ which is a logistic growth model with $r=0.6, K=100$.
 If $V=0$, then $\frac{dP}{dt} = -0.4P$ which is an exponential decay model with solution $P(t) = P(0)e^{-0.4t}$

(b) Set $\frac{dV}{dt} = 0, \frac{dP}{dt} = 0$. From $(-0.4 + 0.005V)P = 0$, either $P=0$ or $(-0.4 + 0.005V) = 0$.
 $0.005V = 0.4 \implies V^* = 80$

From $(0.6(1 - \frac{V}{100}) - 0.02P)V = 0$, either $V=0$ or $0.6(1 - \frac{V}{100}) - 0.02P = 0$.
 Put in $V^* = 80$ and this becomes (using $\frac{80}{100} = 0.8$)
 $(0.6)(0.2) - 0.02P = 0$, from which $P^* = 6$.
 Actually there is another (not so interesting) equil = $V^* = 100, P^* = 0$.