MATH 172 Fall, 2009 Quiz #7 Name: M^2

For full credit you must show sufficient work to justify your answer.

1. Compute the equilibrium point (u^*, v^*) of the discrete model system

$$u_n = 3u_{n-1} - 2v_{n-1} - 4$$
$$v_n = 5u_{n-1} - 3v_{n-1} - 28$$

Set $u^* = u_n = u_{n-1}$
$v^* = v_n = v_{n-1}$

$$\begin{cases} u^* = 3u^* - 2v^* - 4 \\ v^* = 5u^* - 3v^* - 28 \end{cases}$$

$$(-2)\begin{cases} 4 = 2u^* - 2v^* \\ 28 = 5u^* - 4v^* \end{cases}$$

$$\begin{cases} -8 = -4u^* + 4v^* \\ 28 = 5u^* - 4v^* \end{cases}$$

Add up: $20 = u^*$

Then

$2v^* = 2u^* - 4$

$v^* = u^* - 2$

$= 18$

$(u^*, v^*) = (20, 18)$ 2

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2. Consider the following continuous model of a predator-prey system.

$$\frac{dV}{dt} = 0.6V\left(1 - \frac{V}{100}\right) - 0.02VP = \left[0.6\left(1 - \frac{V}{100}\right) - 0.02P\right]V$$
$$\frac{dP}{dt} = -0.4P + 0.005VP = (-0.4 + 0.005V)P$$

Answer: $(V^*, P^*) = (80, 6)$

a. What kind of growth does the victim population exhibit if there are no predators (i.e., $P = 0$)? What kind of long term trend is there for the predator if there are no victims (i.e., $V = 0$)?

b. Compute the equilibrium (V^*, P^*) other than (0,0) for the predator-prey system. Suggestion: find V^* first.

(a) If $P=0$, then $\frac{dV}{dt} = 0.6V(1 - \frac{V}{100})$ which is a logistic growth model with $r = 0.6$, $K = 100$. 1

If $V=0$, then $\frac{dP}{dt} = -0.4P$ which is an exponential decay model with solution 1

$P(t) = P(0)e^{-0.4t}$ 2

(b) Set $\frac{dV}{dt} = 0$, $\frac{dP}{dt} = 0$. From $(-0.4 + 0.005V)P = 0$, ~~either $P=0$ or~~ $(0.4 + 0.005V) = 0$

$.0005V = 0.4$ $V^* = 80$ 1

From $(0.6(1 - \frac{V}{100}) - 0.02P)V = 0$, either $V = 0$ or $0.6(1 - \frac{V}{100}) - 0.02P = 0$

Put in $V = 80$ and this becomes (using $\frac{80}{100} = 0.8$) $(0.6)(0.2) - 0.02P = 0$, from which $P^* = 6.$ 1

Actually there is another (not so interesting) equil: $V^* = 100$, $P^* = 0$.