MATH 172 Fall, 2011 Quiz #6 Name: M^2

1. (15 points) Let $A = \begin{bmatrix} 2 & 5 \\ 6 & 1 \end{bmatrix}$, $\mathbf{u} = \begin{bmatrix} 5 \\ -6 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$, and $\mathbf{w} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$. Which of $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ is an eigenvector, and which is not? Explain, and give the corresponding eigenvalues, where appropriate.

$A\vec{u} = \begin{bmatrix} -20 \\ 24 \end{bmatrix} = -4\begin{bmatrix} 5 \\ -6 \end{bmatrix} = -4\vec{u}$ so $\vec{u}$ is an e-vect with eval -4.

$A\vec{v} = \begin{bmatrix} 13 \\ -3 \end{bmatrix} \neq$ multiple of $\vec{v}$ so $\vec{v}$ is <u>not</u> an e-vect.

$A\vec{w} = \begin{bmatrix} 14 \\ 14 \end{bmatrix} = 7\begin{bmatrix} 2 \\ 2 \end{bmatrix} = 7\vec{w}$ so $\vec{w}$ is an e-vect with eval 7

2. (20 points) A population consists of individuals in four stages of development: newborns (N_t), juveniles (J_t), reproductive adults (R_t), and post-reproductive adults ("grandmothers" G_t). Newborns have a mortality rate of 70%; those that survive become juveniles. Juveniles have a total survival rate of 60% in two categories: 20% remain in the juvenile phase and 40% advance to the reproductive adult phase. Also juveniles rarely reproduce: on average each contributes 1 newborn to the next generation. Reproductive adults have a survival rate of 80% as reproductive adults, and 10% become post-reproductive adults. Meanwhile each contributes 5 newborns to the next generation. Post-reproductive adults produce no offspring, but have a survival rate of 70%.

a. Set up the population transition matrix A to express the information above, so that $\mathbf{P}_{t+1} = A\mathbf{P}_t$.

$$A = \begin{bmatrix} 0 & 1 & 5 & 0 \\ 0.3 & 0.2 & 0 & 0 \\ 0 & 0.4 & 0.8 & 0 \\ 0 & 0 & 0.1 & 0.7 \end{bmatrix}$$

b. The initial population vector is $\mathbf{P}_0 = \begin{bmatrix} 100 \\ 10 \\ 10 \\ 10 \end{bmatrix}$. Compute $\mathbf{P}_1$.

$$\vec{P}_1 = A\vec{P}_0 = \begin{bmatrix} 1(10) + 5(10) \\ (0.3)(100) + (0.2)(10) \\ (0.4)(10) + (0.8)(10) \\ (0.1)(10) + (0.7)(10) \end{bmatrix} = \begin{bmatrix} 60 \\ 32 \\ 12 \\ 8 \end{bmatrix}$$

c. At $t = 10$ we have $\mathbf{P}_{10} = \begin{bmatrix} 883 \\ 238 \\ 185 \\ 30 \end{bmatrix}$ (approximately). Determine the total population and the distribution vector $\mathbf{D}_{10}$. Carry three significant figure accuracy after the "leading" 0's (like 0.0273).

$$\text{Total}_{10} = 1336$$

$$\vec{D}_{10} = \begin{bmatrix} 0.661 \\ 0.178 \\ 0.138 \\ 0.0225 \end{bmatrix}$$

d. The dominant eigenvalue is $\lambda = 1.3147$ with eigenvector $\mathbf{v} = \begin{bmatrix} 0.661 \\ 0.178 \\ 0.138 \\ 0.0225 \end{bmatrix}$.

Has the population reached its stable age/stage distribution at $t = 10$? How can you tell? Use λ to predict the total population at $t = 11$ and the value of N_{11}, that is, the number of newborns at $t = 11$; show your work.

Yes, because the $\vec{D}_{10}$ agrees with the dominant eigenvalue's e-vect $\vec{v}$, which gives the SAD.

$$\text{Total}_{11} \underset{\text{at SAD}}{=} \lambda \cdot \text{Total}_{10} = (1.3147)(1336) = 1756$$

$$N_{11} = \lambda \cdot N_{10} = (1.3147)(883) = 1161$$