

Recall that (1) an affine continuous model $\frac{dQ}{dt} = aQ + b$ has an explicit solution $Q(t) = Ce^{at} + Q^*$, where Q^* is the equilibrium value, and C can be determined from the initial condition, and (2) an affine discrete model $Q_{n+1} = aQ_n + b$ has an explicit solution $Q_n = Ca^n + Q^*$, where Q^* is the equilibrium value, and C can be determined from the initial condition.

1. A population of whales $W = W(t)$ growing at a per capita rate of $2\% \text{ yr}^{-1}$, but 6 migrate away over the course of each year.
- a. Write an affine continuous model equation for this situation, and solve it, assuming that the initial whale population is 250.

$$W' = 0.02W - 6$$

$$0 = 0.02W - 6$$

$$W^* = \frac{6}{0.02} = 300$$

$$W(t) = Ce^{0.02t} + 300$$

$$250 = Ce^0 + 300 = C + 300$$

$$C = -50$$

$$W(t) = 300 - 50e^{0.02t}$$

- b. What exactly happens to the whale population in the long term, and how do you know? If the population is growing, compute the doubling time; if the population is shrinking, compute the extinction time.

Since $e^{0.02t} \rightarrow \infty$ as $t \rightarrow \infty$, more and more is subtracted from 300, so $W(t) \rightarrow 0$. In fact eventually $W(t) = 0$:

$$0 = 300 - 50e^{0.02t}$$

$$e^{0.02t} = \frac{300}{50}$$

$$0.02t = \ln(6) \quad (\text{years})$$

$$t = \frac{\ln(6)}{0.02} = 89.6 \text{ is the time to extinction.}$$

2. The growth rate of a population $N = N(t)$ is governed by the model equation

$$\frac{dN}{dt} = 0.04N \left(1 - \frac{N}{150}\right) \left(\frac{N}{30} - 1\right).$$

a. Determine the equilibrium values.

$$\frac{dN}{dt} = 0 \text{ at } N^* = 0, 150, \text{ and } 30$$

b. If $N(0) = 35$, sketch the long term behavior of $N(t)$, and explain why the graph has the shape that it does. Then do the same if $N(0) = 25$.

If $N(0) = 35$, then

$$N' = (+)(+)(+) > 0$$

and as long as
 $30 < N < 150$,

$N' > 0$, so N is \uparrow

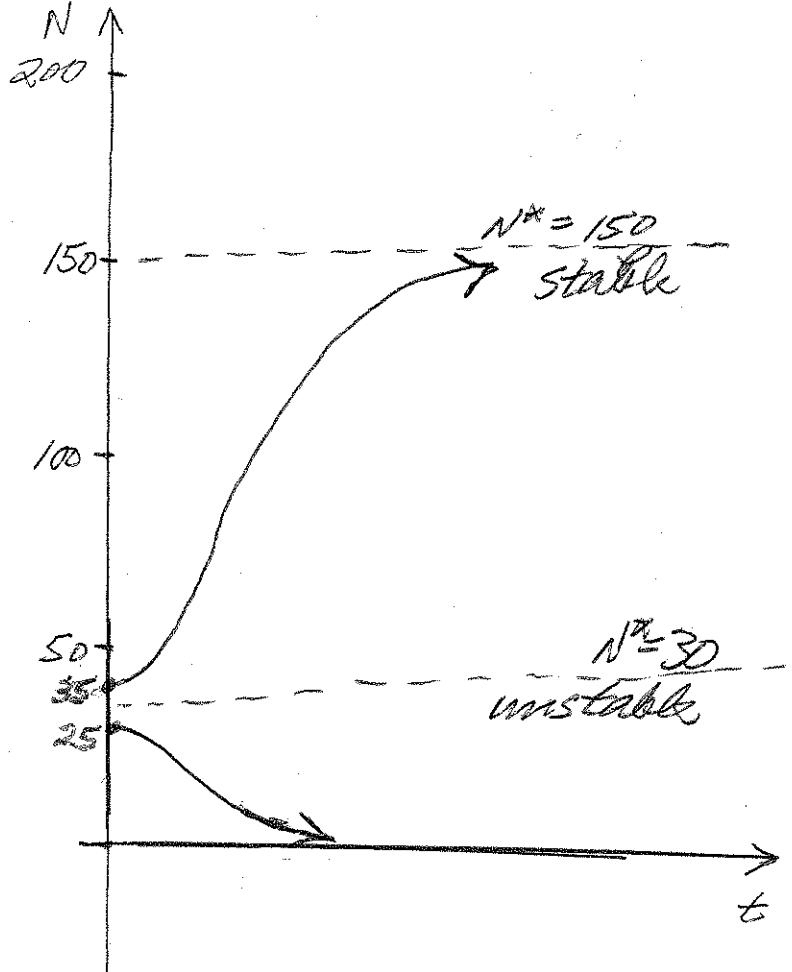
If $N(0) = 25$, then

$$N' = (+)(+)(-) < 0$$

and as long as

$0 < N < 30$,

$N' < 0$, so N is \downarrow



c. Say whether each equilibrium value is stable or unstable.

$N^* = 0$ or 150 stable

$N^* = 30$ unstable