MATH 172 **Fall, 2011** **Quiz #5** **Name:** M^2

Recall that (1) an affine continuous model $\frac{dQ}{dt} = aQ + b$ has an explicit solution $Q(t) = Ce^{at} + Q^*$, where Q^* is the equilibrium value, and C can be determined from the initial condition, and (2) an affine discrete model $Q_{n+1} = aQ_n + b$ has an explicit solution $Q_n = Ca^n + Q^*$, where Q^* is the equilibrium value, and C can be determined from the initial condition.

1. A population of whales $W = W(t)$ growing at a **per capita rate** of 2% yr^{-1}, but 6 migrate away over the course of each year.
   a. Write an affine continuous model equation for this situation, and solve it, assuming that the initial whale population is 250.

$$W' = 0.02W - 6$$
$$0 = 0.02W - 6$$
$$W^* = \frac{6}{0.02} = 300$$
$$W(t) = Ce^{0.2t} + 300$$
$$250 = Ce^0 + 300 = C + 300$$
$$C = -50$$
$$\boxed{W(t) = 300 - 50e^{.02t}}$$

   b. What exactly happens to the whale population in the long term, and how do you know? If the population is growing, compute the doubling time; if the population is shrinking, compute the extinction time.

Since $e^{0.02t} \to \infty$ as $t \to \infty$, more and more is subtracted from 300, so $W(t) \to 0$. In fact eventually $W(t) = 0$:

$$0 = 300 - 50e^{.02t}$$
$$e^{.02t} = \frac{300}{50}$$
$$0.02t = \ln(6) \quad \text{(years)}$$
$$t = \frac{\ln(6)}{0.02} = 89.6$$ is the time to extinction.

2. The growth rate of a population $N = N(t)$ is governed by the model equation
$$\frac{dN}{dt} = 0.04N\left(1 - \frac{N}{150}\right)\left(\frac{N}{30} - 1\right).$$

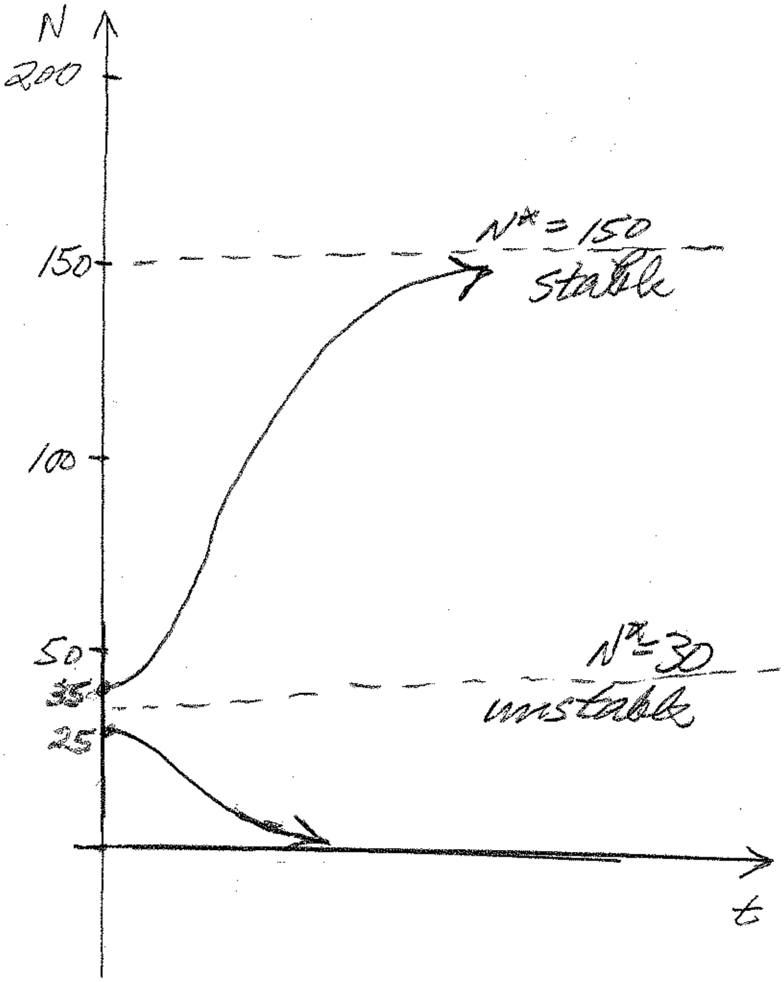
a. Determine the equilibrium values.

$\frac{dN}{dt} = 0$ at $N^* = 0,\ 150,$ and 30

b. If $N(0) = 35$, sketch the long term behavior of $N(t)$, and explain why the graph has the shape that it does. Then do the same if $N(0) = 25$.

If $N(0) = 35$, then
$N' = (+)(+)(+) > 0$
and as long as
$30 < N < 150$,
$N' > 0$, so N is ↑

If $N(0) = 25$, then
$N' = (+)(+)(-) < 0$
and as long as
$0 < N < 30$,
$N' < 0$, so N is ↓



c. Say whether each equilibrium value is stable or unstable.

$N^* = 0$ or 150 stable
$N^* = 30$ unstable