MATH 172 Fall, 2009 Quiz #5 Name: M^2

logistic model

1. The growth rate of a population $P = P(t)$ is governed by the model equation $\frac{dP}{dt} = 0.04P\left(1 - \frac{P}{80}\right)$. Determine all the equilibium values. Sketch the long term behavior of $P(t)$ given each of the initial conditions.

   a. $P(0) = 20$. Label this graph A.

   b. $P(0) = 100$. Label this graph B.

   c. $P(0) = 60$. Label this graph C.

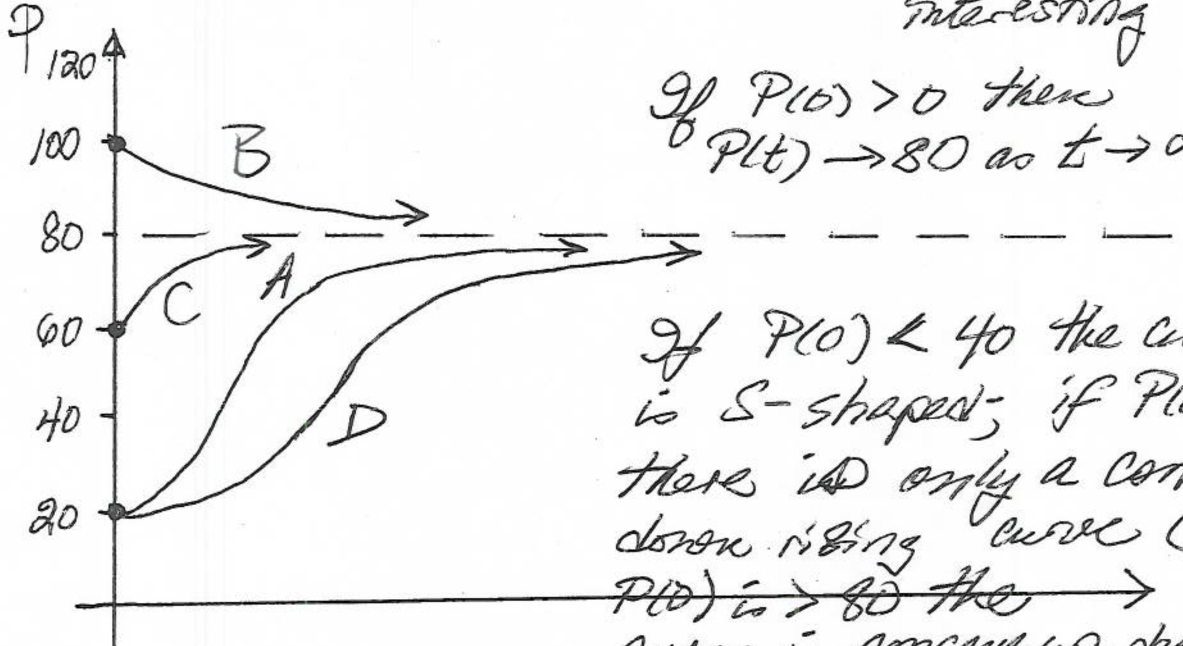
   d. Now suppose 0.04 is replaced by 0.01 in the model. Take $P(0) = 20$ again. Sketch the new graph and label it D.

$0 = 0.04P\left(1 - \frac{P}{80}\right)$

$P^* = 0$ $P^* = 80$

not interesting

carrying capacity



If $P(0) > 0$ then $P(t) \to 80$ as $t \to \infty$.

If $P(0) < 40$ the curve is S-shaped; if $P(0) \geq 40$, there is only a concave down rising curve (if ~~P(0) is > 80 the~~ curve is concave up, decreasing).

2. The growth rate of a population $N = N(t)$ is governed by the model equation $\frac{dN}{dt} = 0.04N\left(1 - \frac{N}{150}\right)\left(\frac{N}{30} - 1\right)$. If $N(0) = 25$, sketch the long term behavior of $N(t)$, and explain why the graph has the shape that it does.

Equil: $N^* = 0$

$N^* = 150$ (carrying capacity)

$N^* = 30$ (threshold)

At $N = 25$, $\frac{dN}{dt} = -0.14$

$N = 15$, $\frac{dN}{dt} = -0.27$

$N = 5$, $\frac{dN}{dt} = -0.16$

If $N < 30$, then $\frac{dN}{dt} < 0$, so $N(t)$ is decr.