MATH 172 **Fall, 2011** **Quiz #4** **Name:** M^2

1. We have $\Delta P = -0.2P_n$.
   a. Write the updating equation.

$$P_{n+1} - P_n = -0.2P_n$$

$$P_{n+1} = P_n - 0.2P_n = 0.8P_n$$

   b. If $P_0 = 50$, write the explicit solution for P_n.

$$P_n = 50(0.8)^n$$

   c. Describe the long term behavior of P_n as $n \to \infty$.

$P_n \to 0$ because $(0.8)^n \to 0$

2. Suppose $Q_{n+1} = -0.7Q_n + 153$.
   a. Compute the equilibrium value Q^*.

$$Q^* = -0.7Q^* + 153$$

$$1.7Q^* = 153 \qquad Q^* = 90$$

   b. If $Q_0 = 100$ determine the solution equation for Q_n. Recall that an affine discrete model $P_{n+1} = aP_n + b$ has an explicit solution $P_n = Ca^n + P^*$, where P^* is the equilibrium value, and C can be determined from the initial condition.

$$a = -0.7$$

$$Q_n = C(-0.7)^n + 90 = 10(-0.7)^n + 90$$

$$100 = Q_0 = C(-0.7)^0 + 90 = C + 90, \quad C = 10$$

   c. Determine the long term behavior of Q_n as $n \to \infty$.

$Q_n \to 90$ oscillating above and below with smaller & smaller oscillations since $(0.7)^n \to 0$ and $(-1)^n$ flip flops sign.