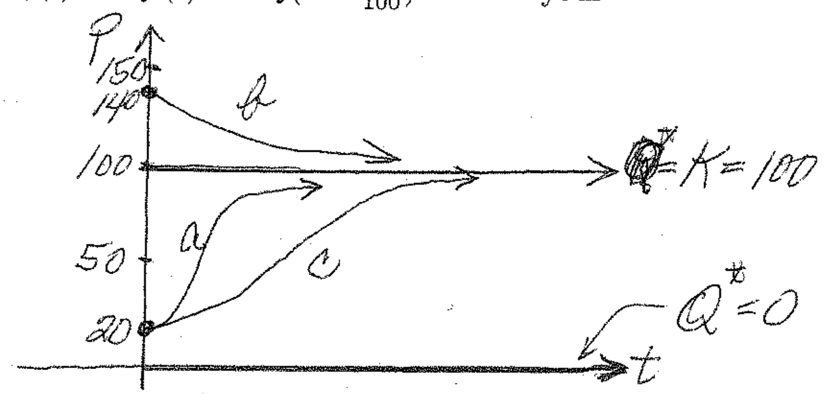
MATH 172 **Fall, 2011** **Quiz #3** Name: MZ

1. (7 points) Sketch all the graphs of $Q(t)$ if $Q'(t) = rQ(1 - \frac{Q}{100})$. Label your graphs clearly with a, b, c.
   a. if $r = 0.8$ and $Q(0) = 20$
   b. if $r = 0.3$ and $Q(0) = 140$
   c. if $r = 0.3$ and $Q(0) = 20$

Equil from $Q' = 0$:

$Q^* = 0, \; Q^* = 100$



2. Due to poor environmental conditions, brook trout are declining continuously by 15% a year. A particular stream is stocked with 5 thousand farm-raised trout a year. Write a model equation for $P(t)$ to describe the net rate of change of the brook trout population over time. Assume also that in our baseline year the population is estimated to be 70 thousand trout. Solve this model equation approximately by using your calculator, using 60 steps to estimate the population after 6 years. Give the value of Δt and the model equation for $u(n)$ that you enter into the calculator. Could they get away with not artificially stocking the stream?

Hint: To get this into your calculator remember to set the MODE on Seq, and use Y= to give the formula for $u(n)$ interms of $u(n-1)$. Set $n\text{Min} = 0$, $u(n\text{Min}) = 70$, and remember that $P(t+\Delta t) = P(t) + \Delta P$, where $\Delta P \approx (\Delta t)P'(t)$ is a good approximation if Δt is small. Now ΔP becomes $\Delta u = u(n) - u(n-1)$, and P or $P(t)$ becomes $u(n-1)$, which is used in the formula for $P'(t)$. End result: $P(t + \Delta t) = P(t) + (\Delta t)P'(t)$ and $u(n) = u(n-1) + (\Delta t) * (P')$, where P' is written in terms of numbers and $u(n-1)$. Make sure that Tblset is OK, then use Table to get the answer.

$P'(t) = -0.15P + 5 \qquad P(0) = 70$

Equil: $P' = 0 = -0.15P + 5, \quad P^* = \frac{5}{0.15} = 33.3$

$\Delta t = \frac{6}{60} = 0.1 \qquad \boxed{n\text{Min} = 0} \quad \boxed{u(n\text{Min}) = 70}$

$P(t+\Delta t) = P(t) + (\Delta t)(-0.15P + 5)$

$u(n) \quad = u(n-1) + (0.1) * (-0.15 * u(n-1) + 5)$

$u(60)$ gives $P(6)$ as $\boxed{48.14}$ thousand trout. Stocking is needed; otherwise $P'(t) = -0.15P$ has solution $P(t) = 70e^{-0.15t} \to 0$ as $t \to \infty$. Actually if run out to 200-300 steps you'll see that long term $P(t) \to P^* = 33.3$, which is maintained by the stocking.