

1. Give the updating equation (also known as the recurrence equation) for the length l_n of a chain of n grocery buggies, where each buggy is 3.5 feet long, and when you push a new buggy into the chain, only 6 inches sticks out. Note that the pattern doesn't really begin until you actually have a buggy, so l_0 is not defined, $l_1 = 3.5$, and $l_2 = 4.0$, $l_3 = 4.5$. Then give the solution equation for l_n in terms of n .

$$6'' = 0.5 \text{ ft}$$

n	l_n	Δl
0		
1	3.5	0.5
2	4.0	0.5
3	4.5	0.5
4	5.0	0.5
...		
n		

$$\left[\text{updating: } l_{n+1} = l_n + 0.5 \right]$$

$$\begin{aligned} l_n &= 3.5 + (n-1)(\Delta l) \\ &= 3.5 + (n-1)(0.5) \\ &= 3.0 + 0.5n \end{aligned}$$

or there is a fictitious $l_0 = 3.0$, and

explicit solution: $l_n = 3.0 + 0.5n$ fits the standard pattern

2. During the 1980's Costa Rica had the highest deforestation rate in the world at 2.9% per year. Deforestation (meaning loss of forested land) is a continuous process.
- a. If $F(t)$ is the amount of forested land, write the model equation for this process.

$$\left[F'(t) = \underset{\substack{\uparrow \\ \text{loss of } F(t)}}}{-0.029} F(t) \quad r = -0.029 \right]$$

- b. Give the explicit solution to this equation.

$$\left[F(t) = F_0 e^{-0.029t} \right]$$

3. Suppose a population $B(t)$ of bacteria is growing over time so that the per capita rate of increase is 0.2% per hour. At the same time 4 mg of the bacteria are withdrawn per hour. Assume that this process take place continuously. (There is a vessel designed for this purpose called a chemostat.)

a. Write the model equation that describes this situation.

$$B'(t) = 0.002 B(t) - 4 \quad \text{mg/L/hr}$$

net rate
gain rate
loss rate

b. Is there a steady state or equilibrium value for the amount of bacteria? If so, compute it.

Set $B' = 0$, solve for B :

$$0 = 0.002 B - 4$$

$$4 = 0.002 B$$

$$B^* = 2000 \text{ mg}$$

* indicates equilibrium

c. What will happen to an initial population of ¹⁰⁰⁰100 mg?

At $B = 1000 \text{ mg}$, $B' = -2 < 0$

Since $B' < 0$, $B(t)$ will decrease.

Then $B'(t)$ will become even more negative, so $B(t)$ will decrease even faster. Eventually $B(t)$ will crash to 0.

Bacteria
+1

