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MATH 172 Fall, 2011 Quiz #1 Name: M^2

1. A population grows so that $P_{n+1} = (1.04)P_n$, where n represents generations; the initial population is $P_0 = 100$.
   a. Compute P_1, P_2, P_3, and the general solution, which is an **equation**, for this model.

1
$$P_1 = 1.04 P_0 = (1.04)(100) = 104$$
$$P_2 = 1.04 P_1 = (1.04)^2 P_0 = 108.16$$
$$P_3 = 1.04 P_2 = (1.04)^3 P_0 = 112.49$$

1
$$P_n = (1.04)^n P_0 = (1.04)^n (100)$$

   b. Compute the population after 100 generations.

1
$$P_{100} = (1.04)^{100}(100) = 5050.49$$

   c. Rewrite the model equation to have the form of a difference equation, that is, some equation of the form $\Delta P =$ something in terms of P_n, where you may take $\Delta P = P_{n+1} - P_n$.

1
$$\Delta P = P_{n+1} - P_n = 1.04P_n - P_n$$
$$= 0.04\,P_n$$

2. Suppose a population $S(t)$ of skinks is growing over time so that the **per capita** rate of increase is 0.007/day. Assume that skinks reproduce continuously (at least through the summer). The initial population is $S(0) = 3000$.
   a. Write the model equation that describes this situation.

1
$$\frac{dS}{dt} \text{ or } S' = 0.007S$$

   d. Show how to get an **approximation** for the skink population in 20 days using just one step. Then do the same using two steps. (Hint: remember $\Delta S \approx S'\Delta t$; what is Δt in each case?)

1 ① $\Delta t = 20$
1 big step

$$\Delta S \approx S'(0)\Delta t = (0.007)(3000)(20)$$
$$= 420$$
$$S(20) = S(0) + \Delta S = \boxed{3420}$$

2 ② $\Delta t = 10$
2 smaller steps

t	$S(t)$	$S'(t)$	$\Delta S \approx S'(t)\Delta t$
0	3000	21	210
10	3210	22.5	225
20	$\boxed{3435}$		

bigger value, more accurate