MATH 172 Fall, 2011 Exam 1 Name: M^2

For full credit you must show your work. All answers should be **rounded to 3 decimal places**, or fewer or more if that is what is provided in the data.

1. (7 points) If $A(n) = 7n - 4n^2 + 1$, compute ΔA.

$$\Delta A = A(n+1) - A(n) = 7(n+1) - 4(n+1)^2 + 1 - (7n - 4n^2 + 1)$$
$$= 7n + 7 - 4n^2 - 8n - 4 + 1 - 7n + 4n^2 - 1$$
$$= -8n + 3$$

you can do these in either order.

2. (10 points) Suppose that $B_t = 300(0.95)^t$ represents a bison population.

a. Compute ΔB in terms of B_t (hint: factor!).

$$\Delta B = B_{t+1} - B_t = 300(0.95)^{t+1} - 300(0.95)^t$$
$$= 300(0.95)^t(0.95 - 1)$$
$$= -0.05 B_t$$

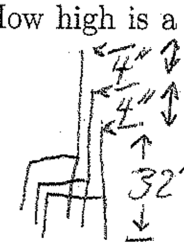
b. Write the underlying discrete model in updating form.

$$B_{t+1} = B_t + (-0.05 B_t) = (1 - 0.05)B_t$$
$$= 0.95 B_t$$

c. What happens to B_t as $t \to \infty$?

Since $(0.95)^t \to 0$ as $t \to \infty$, we have $B_t \to 0$; each year the pop is just 95% of the year before.

3. (10 points) At home I have stacking chairs each of which is 32 inches tall. When one is placed on top of the other 4 inches of the top one sticks out. Write an updating equation for the height h_{n+1} of a stack of $n+1$ chairs in terms of h_n. Give the height of a stack of three chairs and four chairs. How high is a stack of n chairs; *i.e.*, what is h_n in terms of n?



$$h_{n+1} = h_n + 4'' \text{ (inches)}$$
$$h_3 = 32'' + 4'' + 4'' = 40''$$
$$h_4 = 32'' + 3(4'') = 44''$$
$$h_n = 32'' + (n-1)4'' = 28'' + 4''n$$

4. (18 points) The estimated 2007 population of Nicaragua was 5.68 million people, and the net growth rate, assuming continuous increase, was 0.1056 million people per year. Use $N(t)$ for the population at time t taking $t = 0$ to represent the year 2007.

a. Compute the intrinsic (or per capita) growth rate and write the model equation.

$$r = \frac{N'}{N} = \frac{0.1056}{5.68} = 0.0186 \text{ /yr} \quad \boxed{N' = 0.0186\,N}$$

b. Write the explicit solution.

$$N(t) = N_0 e^{rt} = 5.68\, e^{0.0186t} \text{ million people}$$

c. When will the population reach 8 million? t

$$8 = 5.68\, e^{0.0186t}$$

$$\ln\left(\frac{8}{5.68}\right) = 0.0186t \qquad \boxed{t = 18.413} \text{ yr}$$

d. Estimate the population in 2037 by a three-step approximation. In this case $\Delta t =$ 10. Here is the outline of a table to help you get started.
Answer: 9.475 million (falls behind exact value of 9.924)

n	year	t	$N(t)$	$N'(t) = r_N N(t)$	$\Delta N \approx N'(t)\Delta t$	$N(t+\Delta t)$
0	2007	0	5.68	0.1056	1.056	6.736
1	2017	10	6.736	0.1253	1.253	7.989
2	2027	20	7.989	0.1486	1.486	9.475
3	2037	30	9.475	not	needed	→

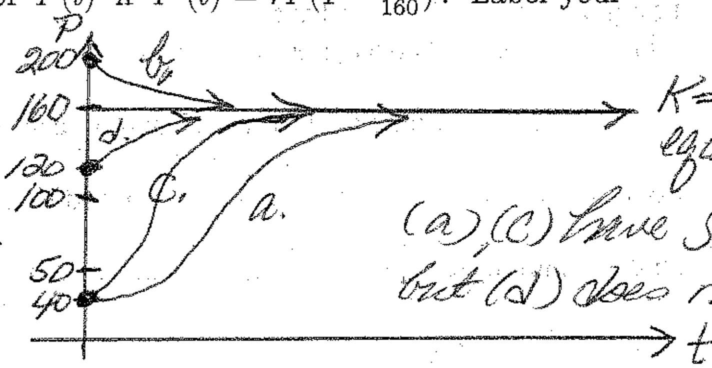
5. (7 points) Sketch all the graphs of $P(t)$ if $P'(t) = rP(1 - \frac{P}{160})$. Label your graphs clearly with a, b, c, d.

a. if $r = 0.3$ and $P(0) = 40$

b. if $r = 0.3$ and $P(0) = 200$

c. if $r = 0.8$ and $P(0) = 40$ — faster per capita growth rate than (a)

d. if $r = 0.3$ and $P(0) = 120$ — starts above $80 = 160/2$



(a), (c) have S-curves, but (d) does not.

e. What is this kind of model called? What is the biological significance of the number 160? of the value of r?

logistic

r = intrinsic or per capita growth rate

$K = 160$ is the "carrying capacity" of the habitat.

6. (15 points) A population of 200 million beetles grows by 3.6% each year (generation). It reproduces and then dies so the new generation replaces the old one once a year.

a. Write the discrete model equation for this process in both difference and updating forms.

$$\Delta B = 0.036 B_n$$

$$B_{n+1} - B_n = 0.036 B_n$$

$$B_{n+1} = 1.036 B_n$$

b. Write the explicit solution equation for this model.

$$B_n = (1.036)^n B_0 = 200(1.036)^n$$

c. How long does it take for the population to reach 300 million?

$$300 = 200(1.036)^n$$

$$\frac{3}{2} = (1.036)^n$$

$$\ln(\tfrac{3}{2}) = \ln((1.036)^n) = n \ln(1.036)$$

$$n = \frac{\ln(3/2)}{\ln(1.036} = 11.46 \text{ yrs or 12 generations}$$

7. (15 points) A region contains 500 thousand acres of wetlands.

a. They are being lost to development at a continuous rate of 8% per year but recovery efforts are producing 28.8 thousand acres per year. Is there an equilibrium value for the area of wetlands? If so, compute it. In the near term what is happening to the area of the wetlands; briefly explain.

$$W'(0) = 28.8 + (-0.08)(500) = -11.2 < 0$$

$$W' = -0.08W + 28.8 \text{ thousand acres/yr}$$

$$0 = -0.08W + 28.8$$

$$W^* = \frac{28.8}{0.08} = 360 \text{ thousand acres}$$

Since $W'(0) < 0$, $W(t)$ is ↓ in the near term.

b. The explicit solution of the model equation is $W(t) = Ce^{-0.08t} + 360$. Show how the initial value of $W(0) = 500$ can be used to compute C. What is the long term trend for the amount of wetlands?

$$500 = W(0) = Ce^0 + 360$$

$$C = 140$$

$$W(t) = 140e^{-0.08t} + 360 \rightarrow 0 + 360 = 360 \text{ as } t \to \infty$$

c. Is the equilibrium stable or unstable? Briefly explain.

Appears to be stable because if $W(t)$ deviates from 360 to 500, $W(t)$ returns to 360 as $t \to \infty$.

8. (18 points) A population of continuously breeding park pigeons has an intrinsic growth rate of 2% per year, but Tom L. is poisoning them causing two deaths per week, or 104 a year.

a. Is there an equilibrium value for the pigeon population? If so, compute it.

$P' = 0.02P - 104$ pigeons/yr

Set $P' = 0$ to find P^*. $0 = 0.02P - 104$

$P^* = \frac{104}{0.02} = 5200$

b. Suppose that there are currently 3000 pigeons. In the near term what is happening to the population; briefly explain.

$P'(0) = (0.02)(3000) - 104 = -44 < 0$

Since $P' < 0$, $P(t)$ is decreasing

c. The explicit solution of the model equation is $P(t) = Ce^{0.02t} + 5200$. Show how the initial value of $P(0) = 3000$ can be used to compute C. What is the long term trend for the pigeon population?

$3000 = P(0) = Ce^0 + 5200$ $\quad e^0 = 1$

$3000 = C + 5200$

$C = -2200$

$P(t) = 5200 - 2200(e^{0.02t})$

∞ as $t \to \infty$, So we are subtracting more & more.

Eventually $P(t) = 0$, and T.L. declares victory.

d. Is the equilibrium stable or unstable? Briefly explain.

Unstable, because as $P(t)$ deviates to 3000 from P^*, $P(t)$ moves even further from P^*.

e. Now suppose that there are initially 6000 pigeons. Use the model equation to describe the short term trend.

$P'(0) = 0.02(6000) - 104 = 16 > 0$

Since $P' > 0$, $P(t)$ is increasing

(again showing instability of $P^* = 5200$)

f. If we were not so fortunate to know the eplicit solution we would have to use an approximation scheme. If there are initially 6000 pigeons, how many will there be after 5 years if we use 20 steps? 6083.9 Hint: To get this into your calculator remember to set the MODE on Seq, and use Y= to give the formula for $u(n)$ interms of $u(n-1)$. In this case

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$\Delta t = \underline{0.25}$. Set $n\text{Min} = 0$, $u(n\text{Min}) = 6000$, and remember that $\Delta P \approx (\Delta t)P'(t)$. Now ΔP becomes $\Delta u = u(n) - u(n-1)$, and P or $P(t)$ becomes $u(n-1)$, which is used in the formula for $P'(t)$. End result: $P(t+1) = P(t) + (\Delta t)P'(t)$ and $u(n) = u(n-1) + (\Delta t) * (P')$, where P' is written in terms of numbers and $u(n-1)$. Make sure that Tblset is OK, then use Table to get the answer. For partial credit on this problem, show what you put into your calculator.

$$u(n) = u(n-1) + (0.25) * (0.02 * u(n-1) - 104)$$

n	P(n)
0	6000
1	6004
2	6008
3	6012.1
4	6016.1
5	6020.2
⋮	
10	6040.9
⋮	
15	6062.1
⋮	
20	6083.9

Clearly P(n) is continually increasing in this case, so T.L. is overwhelmed and concedes defeat.

In fact the explicit solution turns out to be

$P(t) = 800e^{0.02t} + 5200$

which clearly grows to ∞ as $t \to \infty$.

("C" is found by setting $t=0$ in $P(t)$'s formula)