MATH 142 **Exam #3** **Name:**______________________

Spring, 2003 **Lab section by number or by time:** _______________

There are 100 points. **For full credit you must show your work!**

1. (10 points) Give the Maclaurin series for $g(x) = x\sqrt[3]{1+x}$ through degree 4. (Note: that is supposed to be a cube root.)

2. (12 points) **If the Maclaurin polynomial of degree 3 for** $g(x) = e^x$ **is used to approximate** $e^{-0.3}$ **, the result is** 0.7405 (there is no need for you to redo this computation). We previously used the remainder formula to estimate the error. We will now use another method.

   a. Plug $x = -0.3$ into the Maclaurin series for e^x, showing the terms up through the 4^{th} power of -0.3, and the general n^{th} term.

   b. This series converges diverges (circle one) because it is _______________ and $a_n \to$ _______ as $n \to$ _______ and a_n _______ a_{n+1}. By the _______________ Test, $e^{-0.3} \approx 0.7405$ with error at worst _______________ . Briefly indicate the reasons for your answers.

3. (12 points) Compute $\int_0^1 \cos(t^2)\,\mathrm{d}t$ using Maclaurin series methods, obtaining a value accurate within $0.002 = 1/500$. Explain how you know the error is this small, based on an examination of the series.

4. (12 points) On an interval containing 0, we have $f(x) = \sum_{n=0}^{\infty} n^2 3^{2n} x^n$.

   a. Compute $f^{(4)}(0)$. (Hint: this is EASY!)

   b. Compute the interval of absolute convergence of this power series. Do not bother with the endpoints.

5. (54 points) Determine if the series converges, why or why not, or under what conditions on any variable quantity that appears. For the convergent series, give the sum, if possible, or say that the sum is unknown.

a. $\displaystyle\sum_{i=0}^{\infty} \frac{7^i}{3^{i+1}}$

b. $1 - 4t + (4t)^2 - (4t)^3 + (4t)^4 - (4t)^5 + \cdots$

c. $\displaystyle\sum_{k=1}^{\infty} \frac{1}{\arctan(k)}$

d. $\displaystyle\sum_{n=1}^{\infty} \frac{3n}{n^3+1}$

e. Let $S = \dfrac{1}{2} - \dfrac{(1/2)^3}{3!} + \dfrac{(1/2)^5}{5!} - \dfrac{(1/2)^7}{7!} + \cdots + (-1)^n \dfrac{(1/2)^{2n+1}}{(2n+1)!} + \cdots$.

This series converges and the value of S is _______ . Determine if this series is also absolutely convergent.