MATH 142 **Exam #2** **Name:**_____

Spring, 2003 **Lab section by number or by time:** _____

There are 100 points. **For full credit you must show your work!**

1. (54 points) Compute the quantity. In the case of improper integrals or indeterminate forms you must indicate the correct limit argument; you will not get full credit for using ∞ as a number, even if your answer is correct.

   a. $\displaystyle\int \frac{-2x+17}{x^2+3x-4}\,\mathrm{d}x$

   b. $\displaystyle\int_1^{10} \frac{1}{\sqrt{t-1}}\,\mathrm{d}t$

   c. $\displaystyle\int_0^{\infty} \frac{1}{4+y^2}\,\mathrm{d}y$

d. $\displaystyle\int \arctan(3r)\,\mathrm{d}r$

e. $\displaystyle\lim_{x\to\infty} \frac{(\ln x)^2}{\sqrt{x}}$

f. The most reasonable first step in the evaluation of $\displaystyle\int \sqrt{1-e^{6x}}\,\mathrm{d}x$ is to make the substitution(s) _____ _____ , which transform(s) this integral into the following trigonometric integral (but do NOT compute this integral).

2. (16 points) The probability density function for the length of a cell phone call of duration x minutes seems to be $p(x) = 0.17e^{-0.17x}$ for $x \geq 0$ and $p(x) = 0$ for $x < 0$.

   a. What fraction of the calls lasted for 10 minutes or more?

   b. Compute the **median** duration of a call.

   c. (Bonus) Compute the **mean** duration $\mu = \int_0^\infty x\, p(x)\, \mathrm{d}x$.

3. (10 points) Let $f(x) = 1/x$. Compute $P_{4,1}(x)$, the Taylor polynomial of degree 4 centered at $a = 1$.

4. (10 points) If the Maclaurin polynomial of degree 3 for $g(x) = \sin x$ is used to approximate $\sin(1.3)$, the result is 0.93383 (there is no need for you to redo this computation).
   a. In this case we are using $a =$ _____ and $x =$ _____ .
   b. Use the remainder formula to **estimate** how big the error can possibly be. A handy, but not extravagent, overestimate of the error is preferable to one that uses the value $\sin(1.3)$ itself, since that is what you are attempting to approximate in the first place.

   c. For better accuracy we should instead use the Taylor polynomial centered at $a =$ _____ . (What value is closer to 1.3, but where you also know a lot about the values of $\sin x$ and its derivatives?) Bonus: compute this alternate approximation.

5. (10 points) Three species of trees were surveyed in a certain area and the following cumulative distribution functions $P(x)$, $Q(x)$ and $R(x)$ were found, where x was height in meters.

   a. What was the median height for each species? P: _____ Q: _____ R: _____

   b. Which species has the largest fraction of trees that are less than 10 m in height? Explain.

   c. If you pick a tree of type Q at random, what height is it most likely to have? Explain, using a graph of the probability density function for this species.