MATH 122 Fall, 2000 Exam \#1 Name:
There are 100 points. For full credit you must show your work.

1. (33 points) EPA inspectors have taken a sample of murky lake water and placed it in a tube. They shine a light of known intensity at one end of the tube and place a light sensor at various depths down the tube. The depth $D$ is measured in cm and the intensity $I$ is measured as a fraction of full power; here are the results:

| $D$ | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $I$ | .912 | .560 | .344 | .211 | .130 |

a. What is the average rate of change of $I$ from $D=1$ to $D=4$ ?
b. Demonstrate clearly that $I$ can not be a linear function of $D$.
c. Assume that $I$ is a discrete exponential function of $D$ (due to different sediments at different depths). Give a formula for $I$ as a function of $D$. You must use, in one way or another, all the values given in the table.
d. Predict the value of $I$ for $D=3.5 \mathrm{~cm}$ to three decimal places.
2. (10 points) Using the graph of $r=f(p)$, given below, which variable is the dependent variable? $\qquad$ ? Determine the average rate of change (to two decimal places) from $p=0$ to $p=3$ $\qquad$ and from $p=4$ to $p=6$ $\qquad$ . At which value of $p$ is $f(p)$ the greatest? $\qquad$ ?
3. (15 points) The amount of caffeine in a cup of coffee at time $t$ is $A(t)=A_{0} e^{r t}$, where $A_{0}$ is the initial amount. The half-life of caffeine in the body is about 4 hours. What is the "decay rate" $r$ of the caffeine in the body? How long will it take for the level to fall by $75 \%$ of the original amount (hint: what per cent will remain)?
4. (8 points) The carrying capacity $M$ is the maximum number of squirrels that can live on the Horseshoe successfully. The growth rate $G$ of the population of squirrels on the Horseshoe is proportional to the product of the number of squirrels $N$ and the difference between $N$ and the carrying capacity $M$. Write the formula that gives $G$ in terms of $M$ and the present population $N$.
5. (12 points) Assume $s$ is a linear function of $t$, with the following values.

| $s$ | 10 | 6 |  | 0 | -6 | -7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $t$ |  | -2 | 0 | 1 | 4 |  |

a. Which is the independent variable?
b. The slope is $m=$ $\qquad$
c. Fill in the missing values, and find the formula for $s$ as a function of $t$.
d. Write $t$ as a linear function of $s$.
6. (15 points) A company that makes ceiling fans has fixed costs of $\$ 9000$ for a certain product line and variable costs of $\$ 50$ per fan. The company plans to sell these fans for $\$ 80$ each. Let $q$ represent the number of fans. Give formulas for the cost function $C(q)$ and the revenue function $R(q)$. What is the break-even point in terms of number of fans?
7. (7 points) The table below gives the concentration $C(t)$ of carbon dioxide $\left(\mathrm{CO}_{2}\right)$ in parts per million (ppm) in the atmosphere since 1960. Determine and fill in an appropriate scale for $t$. Use your calculator's curve-fitting or regression package to find the best exponential fit for this data, and give the formula. Then use the formula or your graph to estimate the amount of $\mathrm{CO}_{2}$ in the atmosphere the year 2000 .

| year | 1960 | 1965 | 1970 | 1975 | 1980 | 1985 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $t$ |  |  |  |  |  |  |
| $C(t)$ | 316.8 | 319.9 | 325.3 | 331.0 | 338.5 | 345.7 |

