MATH 122 **Second Project (final version)** **Spring, 2004**

Instructions. Choose one of the following problems to work on in a group of **3 or 4** members. These must be **new groups** to the extent possible; **you may have at most one person in your group with whom you worked on the first project**. The reports are due in class on Monday, April 26, but you will receive 5 bonus points if you turn it in on Wednesday, April 21 or earlier–but don't sacrifice quality for these few extra points! I will make every effort to return the earlier ones sooner. I will be glad to offer help up on the project until Thursday, April 22; after that you are on your own. You may not consult with anyone else, and groups may not work together.

You may use any text and your calculators. The reports should reflect a genuine group effort; do not parcel out bits and pieces of the project to different people. Many of the first projects seem to have been written by a single person, and accepted uncritically by the rest–with predictable results. You should write the solution as if it were to be read by an intelligent person, but not necessarily an expert in calculus. This means that a clear formulation of the problem, the ideas and methods used, and the conclusions, must be given in complete English sentences; don't just give a bunch of technical calculations. (In fact, you should suppress tedious arithmetic and algebraic calculations if these don't add to the reader's understanding.) Do NOT make the mistake of thinking that telling which buttons you pushed on the calculator is any kind of explaination–it is not. You must supply adequate mathematical justification for your answers. Where appropriate provide graphs or tables. Make these flow into the text, or if separate at the end, lable them as Table #1 etc., and refer to them this way in the body of your text. Be sure everything is labeled and units are given! Your report should include answers to all the questions that are asked, but not as a checklist. Use them to give you a framework for the things that need to be discussed in the report. Mathematical formulas should be woven into the text, in correct mathematical notation (not computer or caclulator notation), hand written if necessary.

1. You have been asked to advise on the construction of a roller blading rink. The main problem is that the flooring material is very expensive and so it is crucial that the area be calculated precisely. The rink is in the shape of a 32 by 32 yard square with ends formed by parabolas that measure 21 yards from base to vertex (see diagram below). *To find the area of one of the parabolic ends, it will be necessary to introduce a coordinate system and determine the equation of the parabola.* You should be able to quickly determine the coordinates of three points on the parabola, and using this information, to produce an equation for the parabola. With this in hand, *you should be able to calculate the area exactly.* However, to convince the contractor that your answer is really correct, it will also be necessary to give him an answer that, while only approximate, is easier for him to grasp. Give an approximation that supports your exact answer, but that a contractor who does not know calculus can understand (assume he does know basic geometry; after all he is a contractor!)

Next is the problem of purchasing the materials for walls of the rink. This comes down to a question of calculating the perimeter of the rink. One useful piece of information is that if a graph has an equation $y = f(x)$, then then length of the graph (imagining it to be a piece of string that can be straightened out and measured) from $x = a$ to $x = b$ is $\int_a^b \sqrt{1 + (f'(x))^2}\, dx$. Again, you will also need to give an approximation that both helps to check your answer, and that is comprehensible to a contractor who knows only high school geometry, e.g., perimeters of rectangles, triangles, and circles.

2. A cancer research institute has collected data on how long people with a certain kind of cancer survive after a new kind of treatment. It turns out that $F(T) = \int_0^T 0.25e^{-0.25x}\, dx$ gives the fraction of patients that die in the period from 0 up to T years. Another way to say this is that if a patient is selected at random, then $F(T)$ gives the odds that he or she dies before T years are up. Notice that for each value of T, to calculate $F(T)$ you need to compute an integral. For example, if $T = 1$, then $F(1) = \int_0^1 0.25e^{-0.25x}\, dx$. To get an idea how this function behaves, calculate $F(0.5)$, $F(.9)$, $F(1)$, $F(1.1)$, $F(2)$, $F(3)$, $F(5)$, $F(7)$, $F(10)$, etc. Be sure to inform the reader whether you computing these values exactly or approximately; show your work or describe the method. Make a table and plot $F(T)$ as a function of T. What fraction of patients survive for more than 1 year, but die in less than 2 years? (Hint: express this either as a single integral or as a difference of two integrals.) What fraction live for more than 3 years, but less than 6 years? What happens to $F(T)$ as T gets larger and larger? The researchers are very interested in the function $S(T) = 1 - F(T)$. What is the meaning in real life of this quantity? The rule of thumb for evaluating cancer treatment is to look at 5 year survivorship. What fraction of these patients make it to the 5 year year mark, and what fraction doesn't? Now compute $F'(T)$. If possible, first get a formula for $F(T)$ in terms of T. You should then be able to compute a formula for $F'(T)$. If you don't see a quick way to do this, get some ideas by computing $F'(1)$, and then try the general case again. To do this remember what derivatives are all about: use the values of $F(.9)$, $F(1)$, and $F(1.1)$ to get an approximation for $F'(1)$. How can you get a better approximation? What is the meaning of this derivative in real life terms?

The research institute also needs some statistics on the survival of the patients undergoing the experimental treatment. First they want the **median** survival time. This is the time T^* so that half of all patients survive longer than T^* years and half survive less than T^*. In other words they are looking for T^* so that $F(T^*) = \frac{1}{2} = S(T^*)$. Determine this value of T^*. Next they want the **mean**. This is the value of the integral $\int_0^\infty 0.25xe^{-0.25x}\, dx$. Of course you can't literally integrate all the way to ∞, but based on evidence that you can give, you can come up with a pretty reasonable approximation. This quantity is written μ (lower case Greek letter "mu"). Compare the values of T^* and μ. Which is larger? Why is this plausible?

3. If a peach orchard occupies one acre, how many trees should it contain to produce the largest possible crop? An immediate impulse is to plant a lot of trees. But there is a catch: if there are too many trees they will crowd each other and deprive one another of sunlight and nutrients. So the effect of too many trees will be that each tree is individually less productive, and you may be worse off than if you had planted fewer trees. Of course too few trees is no solution either, because one tree can only produce so many peaches, and the extra room may end up wasted. Farmer Tigre Zampa is a pretty smart guy, but he has forgotten his calculus (anyhow he had it a long time ago at some university in the upstate, whose name I forget, and he doesn't remember much besides the parties). He has some information, but he needs your advice. Write up your report so that **both** Farmer Zampa (your client) and I (your boss at Agricultural Advisors, Inc.) can understand it.

When there are very few (say around 5) trees on each acre each tree produces about 800 pounds of peaches in a season; when there are 25 trees each one produces about 700 pounds; when there are 75 trees each one produces about 580 pounds; when there are 100 trees each one produces about 420 pounds; and when there are 150 trees each one produces about 220 pounds; and finally when there are 175 trees each tree produces essentially nothing. Pretty clearly the yield per tree Y depends on x, the number of trees planted per acre. Use the quadratic regression package on your calculator to get a formula for Y in terms of x, and convince Farmer Zampa that your formula that fits the data pretty well by comparing the formula values and the actual data (a table would probably help).

Now use your formula for Y to get a formula for T, the total seasonal production of each acre of Farmer Zampa's orchard, as a function of x (no Y's in the final formula). Do **not** use regression to compute T. You explain to Farmer Zampa why your formula is correct by pointing out how the units of x, Y, and T all fit together consistently. Then you determine what the optimal (best) number of trees to plant will be in order to get the highest possible total production. Of course you also tell him what this production is, just in case he isn't able to use the formula for T himself!

Farmer Zampa is so pleased with your good advice that he hires you to do something else. He is getting $28\frac{1}{2}$ cents a pound in revenue, and you are able to tell him how his revenue $R(x)$ (per acre, for a season) depends on x.

He also hired a consultant just a short time ago who told him that his fixed costs were \$1800; at $x = 30$ the total costs would be \$2800; at $x = 60$ the total costs would be \$5400; at $x = 90$ they would be \$8400; at $x = 120$ they would be \$11200; at $x = 150$ they would be \$11300; and at $x = 180$ they would be back down to \$5000. Once again you use regression to get a good formula for $C(x)$, only this time a statistically oriented friend suggests using the cubic regression package for a better fit. You explain all this to Farmer Zampa (why you needed a formula in the first place, how well the graph fits the data, why it drops off at the end).

Finally, you explain to him how many trees to plant per acre in order to make the most profit (and you even tell him what this profit will be). He is a little skeptical, because this number of trees you now are telling him to plant is not the same as the number you told him to plant to get the highest yield. But you explain how you came to your conclusion, point out what will happen if he foolishly goes for maximum yield instead of taking your advice, and gradually he begins to understand.

4. The text gives projects at the end of chapters 4, 5, 6, and 7. Some, but not all, of these are suitable. If your group would like to do one of these, let me know as soon as possible, and I'll tell you if it is a suitable project or not. The following are approved as alternatives to the problems given above: problems 1 and 2 on pages 217–218, problem 1 on page 250, and problem 1 on page 275.