MATH 122 **Spring, 2012** **Exam 1** **Name:** M 2

There are 100 points. **For full credit you must show your work.**

1. (10 points) You are given that P is a linear function of r. Find the slope m, the P-intercept b. Also fill in the remaining spaces in the table.

P		-6	22	64	92
r	0	1	3	6	8

14   42   28   3   2

$m = \frac{\Delta P}{\Delta r} = \frac{22-(-6)}{3-1} = \frac{28}{2} = 14$

$\Delta P = 42 = m \Delta r$
$= 14 \Delta r$
$\Delta r = 3$

$P = mr + b$ eqn of line:
$-6 = 14(1) + b$
$-20 = b$

$P = 14r - 20$

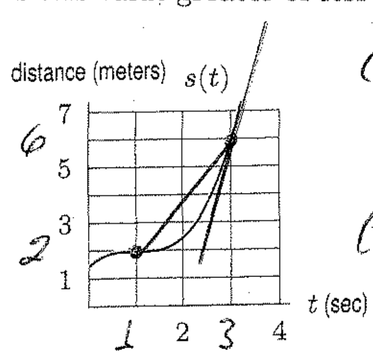
$\Delta P = m \Delta r$
$= 14(2) = 28$
$64 + 28 = 92$

2. (10 points) A company has fixed costs of \$3600 and variable costs of \$40 per unit q. The revenue is \$80 per unit q. Write C and R as functions of q and find the break-even point.

$C = 3600 + 40q$ $R = 80q$

$C = R$ $3600 + 40q = 80q$
$3600 = 40q$
$90 = q$

3. (10 points) The figure gives the distance of a runner from a point.
   a. What is the average rate of change from $t = 1$ to $t = 3$? Give units.
   b. Is this value greater or less than the instantaneous rate of change at $t = 3$?



(a) From $(1,2)$ to $(3,6)$
slope is $\frac{6-2}{3-1} = \frac{4}{2} = 2$ m/sec

(b) Tangent line is steeper than line in (a), so instantaneous rate of change at $t=3$ is greater, avg. rate of change is less.

5

4. (14 points) Could f be a linear function, an exponential function, or is it neither? Briefly explain. Give a formula for $f(x)$, if possible.

x	0	1	2	3
$f(x)$	300	327	356.43	388.51

f can not be linear since slopes are changing.

$\frac{327}{300} = 1.09 \quad \frac{356.43}{327} = 1.09 \quad \frac{388.51}{356.43} = 1.09$ ratio is constant

$f(x) = f_0(1.09)^x = 300(1.09)^x$ exponential

b. Estimate the instantaneous rate of change at $x = 2$, i.e., estimate $f'(2)$.

or use formula from (a) with points close to $x=2$

$\frac{388.51 - 356.43}{3-2} = 32.08$ right slope

$\frac{356.43 - 327}{2-1} = 29.43$ left slope

$f'(2) =$ average $= 30.755$ approx

5. (10 points) What percentage growth rate is represented by $P = P_0(1.06)^t$? How long does it take for the population P to double?

6% growth

$2P_0 = P_0(1.06)^t$

$2 = (1.06)^t$

$\ln 2 = \ln(1.06)^t$
$= t\ln(1.06)$

$t = \frac{\ln 2}{\ln(1.06)} = 11.9$

6. (10 points) A threatened bird population started at 5000. Nine years later the population had shrunk to to 3000. Assuming continuous exponential rate of decline, what is this rate? How many birds are expected to be present after 15 years?

$3000 = 5000e^{kt} = 5000e^{k(9)}$

$\frac{3}{5} = e^{9k}$

$\ln(\frac{3}{5}) = 9k \qquad k = \frac{\ln(\frac{3}{5})}{9} = -0.057$

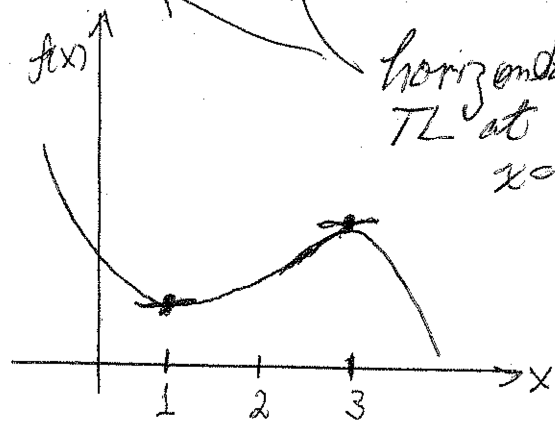
pop after 15 yrs $= 5000e^{(-0.057)(15)} = 2126$

f incr. for $1<x<3$

7. (12 points) Sketch a function whose derivative is positive for $1 < x < 3$, that has $f'(1) = 0$ and $f'(3) = 0$, and that has $f'(x) < 0$ for $x < 1$ and for $x > 3$.

horizontal TL at $x=1$, $x=3$

f decr. for $x<1$, $x>3$



8. (10 points) A donut shop produces $D = f(t)$ donuts at time t, where t is measured in hours from the beginning of their day, and D is measured in dozens of donuts.

a. Interpret the statements $f(6) = 7$ and $f'(6) = -0.4$. Be sure to include the units.

At $t=6$ hours they are producing 7 dozen donuts.
At $t=6$ hours the instantaneous rate of change (marginal production) is -0.4 $\frac{\text{dozen donut}}{\text{hr}}$

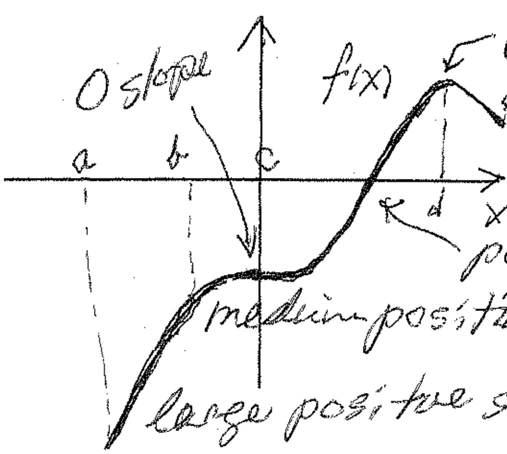
b. Approximately how many dozen donuts are produced at $t = 8$?

$$\Delta D \approx f'(6)\Delta t = (-0.4)(2) = -0.8$$

Production over short time intervals is decreasing at this rate.

$$D(8) \approx D(6) - 0.8 = 7 - 0.8 = 6.2$$

9. (14 points) For the graph given below, sketch the graph of f'.



0 slope

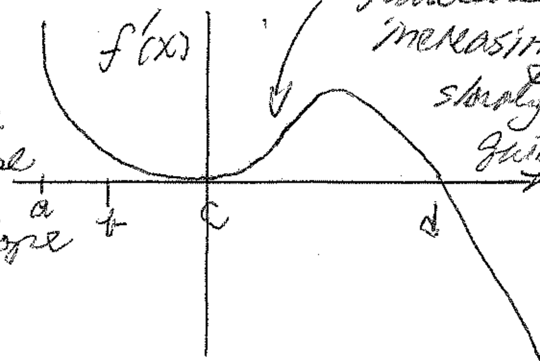
0 slope

negative slope

positive slope

medium positive slope

large positive slope



Between c & d function is increasing: first slowly, then quicker, then slowly again.