11.1 Limits of Sequences

Ex $$\lim_{n\to\infty} \frac{n^2+n}{n^3}$$

$$= \lim_{n\to\infty} \frac{n^2+n}{n^3} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}}$$

$$= \lim_{n\to\infty} \frac{\frac{1}{n} + \frac{1}{n^2}}{1} = \frac{0}{1} = 0$$

Ex: $$\lim_{n\to\infty} \frac{3n^2-1}{n+2n^2}$$

$$= \lim_{n\to\infty} \frac{3n^2-1}{n+2n^2} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}}$$

$$= \lim_{n\to\infty} \frac{3-\frac{1}{n^2}}{\frac{1}{n}+2} = \frac{3-0}{0+2} = \frac{3}{2}$$

<u>Ex</u> $\lim_{n \to \infty} n \sin\left(\frac{1}{n}\right)$

· Replace with a continuous function:

$$= \lim_{x \to \infty} x \sin\left(\frac{1}{x}\right)$$

$$= \lim_{x \to \infty} \frac{\sin\left(\frac{1}{x}\right)}{\frac{1}{x}}$$

· Use L'Hopital's Rule:

$$= \lim_{x \to \infty} \frac{\cos\left(\frac{1}{x}\right) \cdot \left(-x^{-2}\right)}{-x^{-2}}$$

$$= \lim_{x \to \infty} \cos\left(\frac{1}{x}\right)$$

· Since cos is continuous:

$$= \cos\left(\lim_{x \to \infty} \frac{1}{x}\right)$$

$$= \cos(0) = \boxed{1}$$

Ex: $\lim_{n\to\infty} (-1)^n \cdot \frac{1}{n}$

- Use squeeze thm:

$$-\frac{1}{n} \le (-1)^n \cdot \frac{1}{n} \le \frac{1}{n}, \text{ for all } n \ge 1.$$

- $\lim_{n\to\infty} -\frac{1}{n} = 0$

- $\lim_{n\to\infty} \frac{1}{n} = 0$

- So,

$$0 \le \lim_{n\to\infty} (-1)^n \cdot \frac{1}{n} \le 0$$

$$\Rightarrow \lim_{n\to\infty} (-1)^n \cdot \frac{1}{n} = 0.$$