Normal T-Cells (Part 1 of 2)

Step I – Background

Perelson, Kerschner, and DeBoer [4] have presented a model for the interaction of T-cells and HIV. In this project we will examine a preliminary model that explains the normal functioning of T-cells. The model of T-cell formation in the presence of HIV is contained in the project titled "T-Cells in the Presence of HIV".

This project has a slightly different structure than most of the others. The general idea is to explain a biological theory. Mathematical problems will be encountered, formulated, solved, and interpreted throughout this investigation. It might be even more informative if you compared this discussion with the discussion found in the original paper.

Step II – The Basic Model

T-cells are a natural and essential component of the human immune system. The primary job of T-cells is to attack and destroy cancerous cells, cells infected with a virus, and, in general, any cells containing foreign antigens. Prior to examining the interaction between T-cells and HIV, it is necessary to understand a little about how T-cells function under normal conditions.

The typical T-cell count in a healthy person is about 1000 to 1200 per cubic millimeter. T-cells are created in the lymphatic tissues (including the thymus). These tissues normally produce T-cells at a uniform rate. When the body detects a need for T-cells to combat an infection, additional T-cells are created. T-cells only live for a finite period of time.

A mathematical model for the number of T-cells in the body as a function of time (t) must account for the two different mechanisms controlling the production of T-cells and the death of T-cells as they age. Assuming a logistic growth model for the need-based production of T-cells, and a constant per-capita death rate, this model would take the form: $\frac{dT}{dt} = f(T)$ where

$$f(T) = s + rT\left(1 - \frac{T}{T_{max}}\right) - \mu T$$

and the parameters s, r, T_{max}, and μ are all positive.

Step III – Understanding The Basic Model

Identify each of the three terms in the RHS of this differential equation and explain the meaning of each of the parameters r, s, T_{max}, and μ. (What are units associated with each of the parameters? Assume time is measured in days.)

Explain why the conditions $f(0) > 0$ and $f(T_{max}) < 0$ assure that the number of T-cells will stay between 0 and T_{max}. Under what additional constraints on the parameters are these conditions satisfied?

The fact that f is continuous on $[0, T_{max}]$ and changes sign on this interval implies, by the Intermediate Value Theorem, that there is some value of T, say T^*, for which $f(T^*) = 0$.

What is the significance of this number of T-cells? Find T^* in terms of the parameters r, s, T_{max}, and μ.

Step IV – Qualitative Analysis for a Specific Case

Realistic value of the parameters for a healthy individual are $r = 0.03$, $s = 10$, $T_{max} = 1700$, and $\mu = 0.02$.

Do these parameters satisfy the constraints identified earlier? What is T^*? Is this consistent with the biological descriptions?

Determine the qualitative behavior of solutions for all initial conditions between 0 and T_{max} for the specific values of the parameters given. Do all solutions stay in the desired interval?

Step V – Qualitative Analysis for the General Case

Repeat Step II for general values of the parameters.

Bibliography

[1] Greene, W.C., *AIDS and the Immune System*, Scientific American, September 1993 (Special Issue).

[2] Herod, J.V., *A Model for an HIV Infection*, Maple Technical Newsletter, **10**, 1993, pp. 72–78.

[3] Herod, J.V., E.K. Yeagers, *Mathematics, Biology, and Maple*, Maple Technical Newsletter (Special Issue), 1994, pp. 69–76.

[4] Perelson, A.S., D.E. Kerschner, R. DeBoer, *Dynamics of HIV Infections of $CD4_+$ T-Cells*, Mathematical Biosciences, **114**, 1993, pp. 81–125.