Design of an Electrical Network

Step I – Understanding the Problem

The objective of this project is to design an electrical network to meet specific design criteria. The network consists of two loops. The left loop contains a 12volt emf and a 1henry inductor and the right loop contains a $\frac{1}{4}$farad capacitor and 6ohm resistor. A resistor with an unspecified resistance r is on the common side of both circuits (see the accompanying figure). There is no current in the network until a switch, in the left loop, is closed.

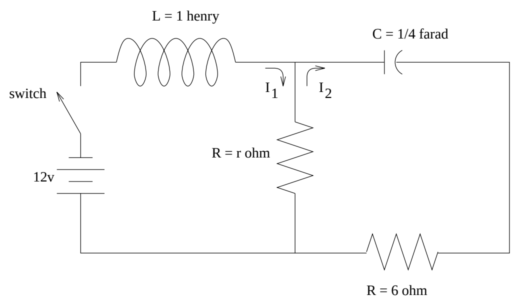

Figure 1: Two-loop network with unspecified common resistance.

The objectives are as follows:

- to determine the steady-state currents (as a function of the unknown resistance, r)
- to identify the different qualitative behavior of the transient currents
- to investigate the length of time required for the currents to converge to within 10% of the steady-state currents

Step II – Mathematical Formulation/Modeling

Denote the current in the left and right loops by $I_1(t)$ and $I_2(t)$, respectively. First-order differential equations for these currents can be obtained by applying Kirchoff's voltage law (§5.5, Nagle/Saff) to each loop of the network.

Write Kirchoff's voltage laws for each loop. Manipulate the equations until they have the (standard) form:

$$\begin{bmatrix} I_1'(t) \\ I_2'(t) \end{bmatrix} = A \begin{bmatrix} I_1(t) \\ I_2(t) \end{bmatrix} + b$$

where A is a 2×2 matrix and b is a two-dimensional vector. (Both A and b will depend on the parameter r.)

(Note that the system is first-order only because neither loop contains both a capacitor and an inductor. How would the equations change if either loop contained both an capacitor and an inductor?)

Since there is no current in either loop prior to the closing of the switch, the initial conditions are $I_1(0) = I_2(0) = 0$.

Step III – Solution and Analysis

Determine the steady-state solution for all resistances $r > 0$.

Determine, for all $r > 0$, a basis of solutions for the homogeneous equation. Describe the trajectories in the phase plane. (Hint: For what values of r do the solutions oscillate? Does A have a full set of linearly independent eigenvectors for all $r > 0$?)

The phase plane analysis should show that all solutions to the homogeneous equation are transient (for all $r > 0$). This analysis does not give any indication of how long the transient solution takes to become decay to a point where it is not significant.

Determine, as a function of r, the amount of time required to ensure that the current in each loop is with 5% of the steady-state current.

Step IV – Summary and Conclusion

This network considered in this project is not very sophisticated. While the problem would involve more variables and the manipulations would be more difficult for more complicated networks, the same analytical methods could be used to determine answers to the same set of questions.

One additional question that is reasonable to consider is the maximum current that passes through the network. For example, suppose the network is rated to support a maximum current of I_{max}amps. What resistances, r, will guarantee that the current in the network never exceeds this limit?