Direction Fields (§1.3)

DIRECTION FIELDS (Section1.3)

```
> restart;
> with( plots ):
> with( DEtools ):
```

Direction fields are a simple, pictorial, technique for understanding how solutions to a differential equation will behave, WITHOUT COMPUTING THE SOLUTION. Direction fields are particularly useful when you first look at a differential equation. Direction fields can be useful when checking if a solution to a differential equation is correct: the tangent lines to the solution must be parallel to the arrows in the direction field. (This works both for symbolic solutions and for numeric solutions.)

Objectives:

- understand the graphical information conveyed by a differential equation
- begin to understand the concept of a solution to a differential equation
- illustrate the use of Maple's `DEplot` command

New Maple Commands:

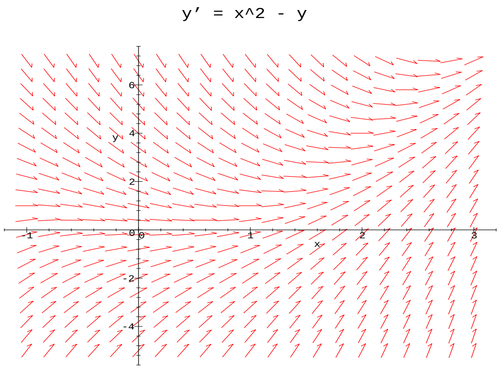
`DEplot` – produces a plot of a direction field for an ODE

The `DEplot` command, in the `DEtools` package, can be used to produce different types of plots for solutions of differential equations. Our most common use will be to create a plot of the direction field for an ODE. Here is how that might look:

```
> ODE := diff( y(x), x ) = x^2 - y(x);
```

$$ODE := \frac{\partial}{\partial x} \mathrm{y}(x) = x^2 - \mathrm{y}(x)$$

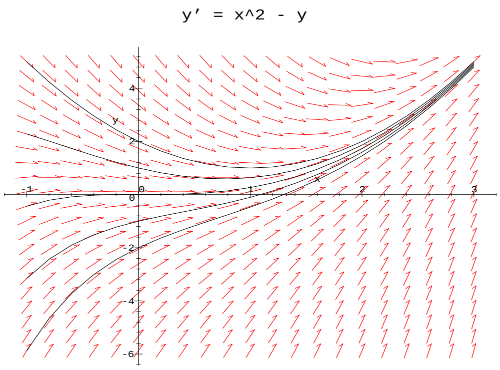
```
> DEplot( ODE, [x,y], x=-1..3, y=-5..7, arrows=THIN, title=`y' = x^2 - y` );
```



- What do you see in this plot?
- What do the arrows indicate?
- How would a stick float down a stream with these flow patterns?
- Does this depend upon where the stick enters the flow?

One way to answer the last question is to use `DEplot` with slightly different arguments.

```
> DEplot( ODE, [x,y], x=-1..3, { [0,-2], [0,-1], [0,0], [0,1], [0,2] },
>          arrows=THIN, title=`y' = x^2 - y` );
```



Notice that the only change is the replacement of the range for the dependent variable (y) with a set of five points. Notice that each of the five curves in the plot passes through a different point in this set.

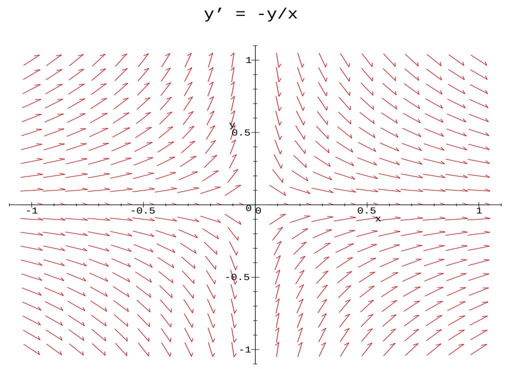
Do you need to reconsider any of your answers to the previous questions? Can you now add more detail to your answers?

Consider the equation $y' = -y/x$:

```
> ODE := diff( y(x), x ) = -y(x)/x;
```

Does this differential equation satisfy the continuity conditions necessary to apply the Existence and Uniqueness Theorem (Section 1.2)? Explain. Can this be seen in the direction field?

```
> DEplot( ODE, [x,y], x=-1..1, y=-1..1, arrows=THIN, title=`y' = -y/x` );
```



EXAMPLE 1 (p. 19)

The logistic equation for a population with size p (in thousands) at time t is

```
> ODE := diff( p(t), t ) = p(t) * ( 2 - p(t) );
```

$$ODE := \frac{\partial}{\partial t}\,\mathrm{p}(\,t\,) = \mathrm{p}(\,t\,)\,(\,2 - \mathrm{p}(\,t\,)\,)$$

Use a direction field to answer the following questions:

- If the initial population is 3000 (that is, $p(0) = 3$), what can you say about the limiting population $\lim_{t\to\infty} p(t)$?
- Can a population of 1000 ever decline to 500? (vice versa?)
- Can a population of 1000 ever increase to 3000? (vice versa?)

```
> DEplot( ODE, [t,p], t=0..5, p=0..4, arrows=THIN,
>          title = `p' = p ( 2 - p )` );
```