Laplace Transforms

laplace.mws

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1 Overview

The Laplace transform is a very common, and useful, technique for solving and analyzing the solution of initial value problems for linear and constant coefficient equations and systems. The key property of the Laplace transform is that derivatives are transformed into algebraic powers of the transform variable; thus, the differential equation is transformed into an algebraic equation. The focus of this worksheet is to understand the Laplace transform, and the inverse Laplace transform, more than the actual solution of a differential equation.

2 Objectives

- to understand the definition of the Laplace transform and the inverse Laplace transform
- to appreciate the general applicability of Laplace transforms to the solution of linear ODEs (both single equations and systems)
- to understand and be able to work with piecewise defined functions
- to become familiar with several of the different ways in which Maple can deal with Laplace transforms

3 New Maple Commands

- alias create an alternate name by which an expression can be referenced
- completesquare complete the square of a polynomial of degree 2 (in student package)

- convert(..., parfrac, s) compute the partial fraction decomposition of an expression

- convert(..., piecewise) convert from Heaviside functions to piecewise-defined functions

- dsolve(..., method=laplace) use Laplace transforms to solve a differential equation

- Heaviside the Heaviside step function (zero if $x<0$, one if $x>0$, and undefined for $x=0$)

- invlaplace apply the inverse Laplace transform (in inttrans package)

- laplace apply the Laplace transform (in inttrans package)

- piecewise create a piecewise-defined "function"

3.1 Optional

- about check assumptions for a variable

- assume provide additional information or constraints on a variable

4 Preliminary Preparations

```
> restart;
```

The full implementation of the Laplace transform is available only after the inttrans library is loaded.

```
> with( inttrans );
```

[*addtable*, *fourier*, *fouriercos*, *fouriersin*, *hankel*, *hilbert*, *invfourier*, *invhilbert*, *invlaplace*, *laplace*, *mellin*]

Likewise, the student package is needed if it is desired to complete the square on an expression (using the completesquare command).

```
> with( student );
```

$[D, \mathit{Diff}, \mathit{Doubleint}, \mathit{Int}, \mathit{Limit}, \mathit{Lineint}, \mathit{Product}, \mathit{Sum}, \mathit{Tripleint}, \mathit{changevar}, \mathit{combine}, \mathit{completesquare}, \mathit{distance}, \mathit{equate}, \mathit{extrema}, \mathit{integrand}, \mathit{intercept}, \mathit{intparts}, \mathit{isolate}, \mathit{leftbox}, \mathit{leftsum}, \mathit{makeproc}, \mathit{maximize}, \mathit{middlebox}, \mathit{middlesum}, \mathit{midpoint}, \mathit{minimize}, \mathit{powsubs}, \mathit{rightbox}, \mathit{rightsum}, \mathit{showtangent}, \mathit{simpson}, \mathit{slope}, \mathit{trapezoid}, \mathit{value}]$

5 The Laplace Transform

Definition 1: The Laplace transform of a function f is defined to be the function F given by the improper integral

$$\mathrm{F}(s) = \int_0^\infty \mathrm{f}(t)\, e^{(-s\,t)}\, dt$$

for all values of s for which the integral exists.

5.1 Examples using the definition

The definition of the Laplace transform can be used to find the transform of

5.1.1 the constant function $\mathrm{f}(t) = 1$