Demonstration of Maple for ODEs

demo-odes.mws

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1 Introduction

It is a good habit to remember to use restart when starting each new problem.

```
> restart;
```

Another good habit is to load the plots and DEtools packages at the top of all worksheets, just in case they will be needed at some point in the analysis.

```
> with( plots );
```

[*animate*, *animate3d*, *changecoords*, *complexplot*, *complexplot3d*, *conformal*, *contourplot*, *contourplot3d*, *coordplot*, *coordplot3d*, *cylinderplot*, *densityplot*, *display*, *display3d*, *fieldplot*, *fieldplot3d*, *gradplot*, *gradplot3d*, *implicitplot*, *implicitplot3d*, *inequal*, *listcontplot*, *listcontplot3d*, *listdensityplot*, *listplot*, *listplot3d*, *loglogplot*, *logplot*, *matrixplot*, *odeplot*, *pareto*, *pointplot*, *pointplot3d*, *polarplot*, *polygonplot*, *polygonplot3d*, *polyhedraplot*, *replot*, *rootlocus*, *semilogplot*, *setoptions*, *setoptions3d*, *spacecurve*, *sparsematrixplot*, *sphereplot*, *surfdata*, *textplot*, *textplot3d*, *tubeplot*]

```
> with( DEtools );
```

[*DEnormal*, *DEplot*, *DEplot3d*, *Dchangevar*, *PDEchangecoords*, *PDEplot*, *autonomous*, *convertAlg*, *convertsys*, *dfieldplot*, *indicialeq*, *phaseportrait*, *reduceOrder*, *regularsp*, *translate*, *untranslate*, *varparam*]

2 Direction Fields and Graphical Solutions

Many introductory courses begin by developing the student's understanding about what a differential equation is, what it means for a function to solve an ODE, and how to perform some analysis directly from the differential equation. Graphical methods are commonly employed in these discussions. The Maple command DEplot, from the DEtools package, provides a comprehensive interface for most graphical needs.

To begin, consider the (linear) differential equation

```
> ODE := diff( y(x), x ) = x^2 - y(x);
```

$$ODE := \frac{\partial}{\partial x}\,\mathrm{y}(x) = x^2 - \mathrm{y}(x)$$

Note that this is the first example used in Section 1.3 of Nagle and Saff. The next few commands reproduce a few of the figures displayed on p. 17.

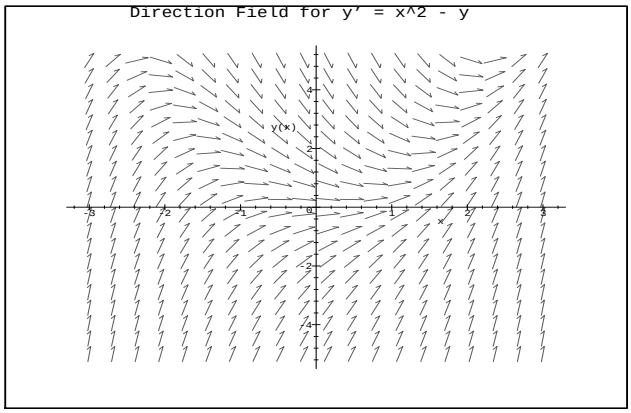
The first three arguments to DEplot provide, in order, a single n-th order ODE or a system of n first-order ODEs, a list indicating the names of the dependent variable(s), and a range for the independent variable. The specific content of the plot is determined by all subsequent arguments to DEplot (see ?DEtools,DEplot for a full description of DEplot).

The most basic use of DEplot is to display a direction field for a differential equation. A request for a direction field is made by specifying both a range for the independent variable and the arrows= option.

2.1 Note about Release 4 Compatibility

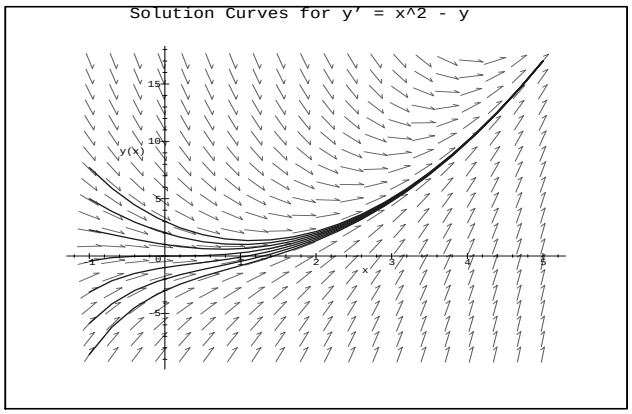
The arrow names have changed between Release 3 and Release 4. Valid names in Release 4 are SMALL, MEDIUM, LARGE, LINE, or NONE.

```
> DEplot( ODE, y, x=-3..3, y=-5..5, arrows=SMALL,
> title=`Direction Field for y' = x^2 - y` );
```



The solution curves through specified points are produced when the fourth argument is a set of initial conditions. If the arrows= argument is included, the curves are superimposed over the arrows; if this argument is omitted, only the solution curves are displayed.

```
> IC := { [0, -3], [0, -2], [0, -1], [0, 0], [0, 1], [0, 2], [0,
3] }:
> DEplot( ODE, y, x=-1..5, IC, arrows=SMALL, color=ORANGE, linecolor=BLUE,
> title=`Solution Curves for y' = x^2 - y` );
```



Notice how the colors of the arrows and solution curves can be controlled separately. Fancier coloring schemes are discussed in the on-line help and later examples.

Only slight alterations are needed for higher-order equations, or for systems. To illustrate, consider the predator-prey system

```
> SYS := [ diff( P(t), t ) = p(t) * P(t) - P(t),
> diff( p(t), t ) = 2 * p(t) - 2 * P(t) * p(t) ];
```

$$SYS := [\frac{\partial}{\partial t}\,\mathrm{P}(t) = \mathrm{p}(t)\,\mathrm{P}(t) - \mathrm{P}(t),\ \frac{\partial}{\partial t}\,\mathrm{p}(t) = 2\,\mathrm{p}(t) - 2\,\mathrm{p}(t)\,\mathrm{P}(t)]$$

where P denotes the size of the predator population and p is the prey.

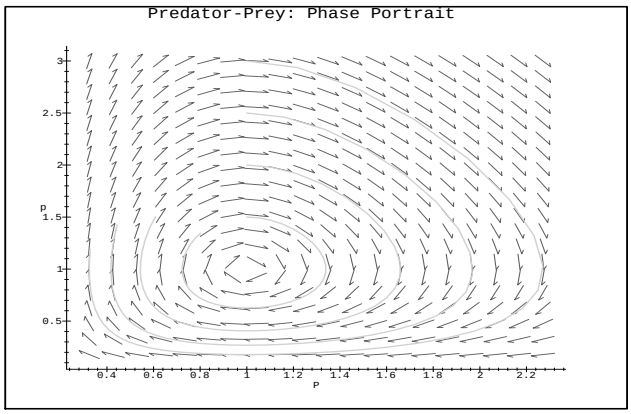
A sampling of solution curves in the phase-space can be created by specifying a set of initial conditions and the desired scene; stepsize=0.1 is included because the default stepsize produces a rather rough graph.

(The use of $ to generate the initial data can be a real time-saver. If this is new to you, please see ?$ for an explanation and additional examples.)

```
> IC := { [0,1,k/2] $ k=2..6 };
```

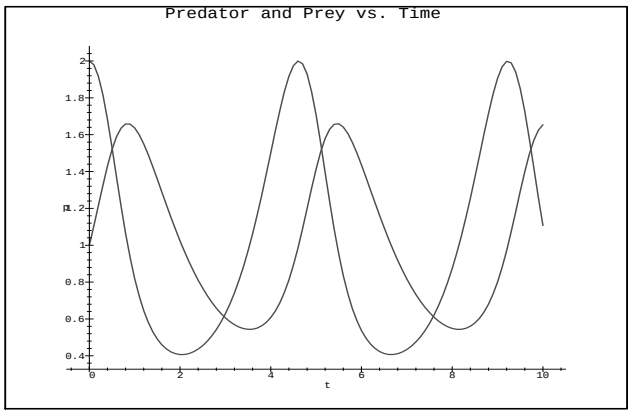
$$IC := \{[0,\ 1,\ 1],\ [0,\ 1,\ 2],\ [0,\ 1,\ \frac{3}{2}],\ [0,\ 1,\ 3],\ [0,\ 1,\ \frac{5}{2}]\}$$

```
> DEplot( SYS, [P,p], t=0..4, IC, scene=[P,p], stepsize=0.1,
> title=`Predator-Prey:  Phase Portrait` );
```



Plots of the individual solutions can be obtained by changing the scene. Here, individual plots of the predator and prey are created, then displayed in a single plot using the display command from the plots package. Note the use of different styles (linecolor in Release 4) to identify the different curves in the plot.

```
> plotp := DEplot( SYS, [P(t),p(t)], 0..10, { [0,1,2] }, scene=[t,p],
> stepsize=0.1, linecolor=BROWN, arrows=NONE,
> title=`Prey vs.  Time` ):
> plotP := DEplot( SYS, [P(t),p(t)], 0..10, { [0,1,2] }, scene=[t,P],
> stepsize=0.1, linecolor=RED, arrows=NONE,
> title=`Predator vs.  Time` ):
> display( [ plotp, plotP ], title=`Predator and Prey vs.  Time`
);
```



2.2 Potential Bug in Release 4

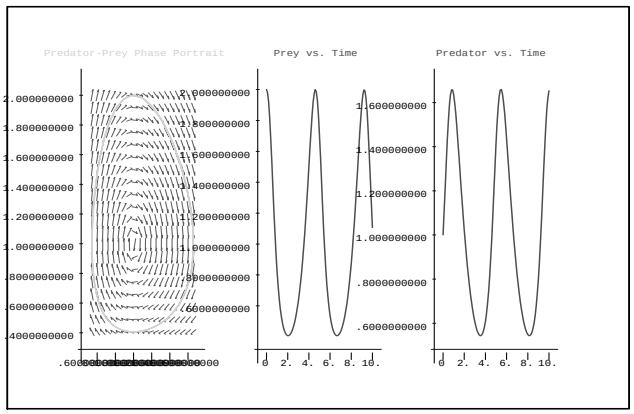
In some cases it is not possible to draw the slope field. Since these cases are not accurately detected by Maple, the default setting of the arrows option causes an error. This has been reported to Maple's Technical Support and has been fixed, as of 5 Nov 1996, in the research version of Maple. Once the fix is incorporated into a patch, this will cease to be a problem. If you do encounter this problem, one possible solution is to explicitly turn off the direction field by specifying arrows=NONE. (This was used in the previous example.)

Note that when the dependent variable is explicitly referenced in the second argument, it can be omitted from the range in the third argument.

2.3 New Feature in Release 4

A new feature in Release 4 is the ability to display an array of plots (see, *e.g.*, the examples in ?plots,display). Here is one way in which this can be used in the predator-prey example.

```
> phaseportrait := DEplot( SYS, [P,p], t=0..10, { [0,1,2] }, scene=[P,p],
> stepsize=0.1, title=`Predator-Prey Phase Portrait` ):
> display( array( [phaseportrait, plotp, plotP] ) );
```



2.3.1 Note about improving the quality of Maple plots

The original version of the above plot may not be very impressive. The quality can be significantly improved by enlarging the region Maple reserves for the plot. To do this, click the left mouse button anywhere in the plot, then select and drag (horizontally) a window anchor to enlarge the plot.

This is only the beginning of what can be done with DEplot and the DEtools package. Please consult the on-line help for additional information for a complete description of the available options and examples.

3 Symbolic Solutions to ODEs and IVPs

While Maple is quite happy to accept the inputs to its commands in a variety of forms, it is practically, esthetically, and pedagogically preferable to use descriptive names for the individual parts of a problem, (*e.g.*, the differential equation and boundary condition). To illustrate, let's find the general solution to

$$x'' + 4\,x' + 4\,x = 2\,t\,e^{(-2\,t)}.$$

The differential equation can be entered as

```
> ODE := diff( x(t), t$2 ) + 4*diff( x(t), t ) + 4*x(t) = 2*t*exp(-2*t);
```

$$ODE := (\frac{\partial^2}{\partial t^2}\,\mathrm{x}(t)) + 4\,(\frac{\partial}{\partial t}\,\mathrm{x}(t)) + 4\,\mathrm{x}(t) = 2\,t\,e^{(-2\,t)}$$

3.1 Note about derivative notation

Note that all derivatives are displayed using partial derivatives.

The general solution is found using dsolve:

```
> GSOLN := dsolve( ODE, x(t) );
```

$$GSOLN := \mathrm{x}(t) = \frac{1}{3}\,t^3\,e^{(-2\,t)} + _C1\,e^{(-2\,t)} + _C2\,t\,e^{(-2\,t)}$$

Note that Maple introduces the constants _C1 and _C2 as arbitrary constants.

The solution to an initial value problem can be obtained by including initial conditions in a set containing the differential equation. For example, the initial conditions $\mathrm{x}(1) = 0$, $x'(1) = 1$ can be implemented as

```
> IC := x(1)=0, D(x)(1)=1;
```

$$IC := \mathrm{x}(1) = 0,\ \mathrm{D}(x)(1) = 1$$

Note the use of D in the specification of the derivative condition (higher order derivative conditions can be specified using @@, Maple's composition operator, *e.g.*, (D@@2)(x)(0)=3. Then, the initial value problem is

```
> IVP := { ODE, IC };
```

$$IVP := \{(\frac{\partial^2}{\partial t^2}\,\mathrm{x}(t)) + 4\,(\frac{\partial}{\partial t}\,\mathrm{x}(t)) + 4\,\mathrm{x}(t) = 2\,t\,e^{(-2\,t)},\ \mathrm{x}(1) = 0,\ \mathrm{D}(x)(1) = 1\}$$

and its solution is found using dsolve as follows:

```
> SOLN := dsolve( IVP, x(t) );
```

$$SOLN := \mathrm{x}(t) = \frac{1}{3}\,t^3\,e^{(-2\,t)} + \frac{1}{3}\,\frac{(2\,e^{(-2)} - 3)\,e^{(-2\,t)}}{e^{(-2)}} - \frac{(e^{(-2)} - 1)\,t\,e^{(-2\,t)}}{e^{(-2)}}$$

Or, in a simpler form:

```
> simplify( SOLN );
```

$$\mathrm{x}(t) = \frac{1}{3}\,t^3\,e^{(-2\,t)} + \frac{2}{3}\,e^{(-2\,t)} - e^{(2-2\,t)} - t\,e^{(-2\,t)} + t\,e^{(2-2\,t)}$$

4 Expressions and Functions

Notice how the output from dsolve is a Maple equation (the reason for this choice will be apparent when considering the solution to a system of differential equations). While there are specific cases where these equations are useful, most circumstances call for either an expression or a function. The key is to understand the structure of the output from dsolve and how to convert it into other forms.

Let's illustrate with the same example as above, with initial conditions specified so that the constants refer to the values of the solution and its first derivative at t=1.

```
> IC := { x(1)=c1, D(x)(1)=c2 };
```

$$IC := \{\mathrm{x}(1) = c1\,,\ \mathrm{D}(x)(1) = c2\}$$

Note that these initial conditions are specified as a set, not an expression sequence. Thus, different manipulations are required to express the final IVP as a set.

```
> IVP := { ODE } union IC;
```

$$IVP := \{\mathrm{x}(1) = c1\,,\ \mathrm{D}(x)(1) = c2,\ (\frac{\partial^2}{\partial t^2}\,\mathrm{x}(t)) + 4\,(\frac{\partial}{\partial t}\,\mathrm{x}(t)) + 4\,\mathrm{x}(t) = 2\,t\,e^{(-2\,t)}\}$$

It is easiest to obtain the solution as an expression; it's just the right-hand side of the object Maple returns from dsolve.

```
> X := simplify( rhs( dsolve( IVP, x(t) ) ) );
```

$$X := \frac{1}{3}\,t^3\,e^{(-2\,t)} + \frac{2}{3}\,e^{(-2\,t)} - e^{(2-2\,t)}\,c1 - e^{(2-2\,t)}\,c2 - t\,e^{(-2\,t)} + 2\,t\,e^{(2-2\,t)}\,c1 + t\,e^{(2-2\,t)}\,c2$$

```
> X := collect( X, {c1,c2} );
```

$$X := (2\,t\,e^{(2-2\,t)} - e^{(2-2\,t)})\,c1 + (t\,e^{(2-2\,t)} - e^{(2-2\,t)})\,c2 + \frac{1}{3}\,t^3\,e^{(-2\,t)} + \frac{2}{3}\,e^{(-2\,t)} - t\,e^{(-2\,t)}$$

When a Maple function is desired, it is most expedient to use unapply.

```
> XX := unapply( X, t );
```

$$XX := t \rightarrow (2\,t\,e^{(2-2\,t)} - e^{(2-2\,t)})\,c1 + (t\,e^{(2-2\,t)} - e^{(2-2\,t)})\,c2 + \frac{1}{3}\,t^3\,e^{(-2\,t)} + \frac{2}{3}\,e^{(-2\,t)} - t\,e^{(-2\,t)}$$

4.1 Personal Pedagogical Note

The use of unapply can be difficult for students to understand; personally, I prefer to avoid this whenever possible.

To illustrate a potential advantage of the use of unapply, consider a situation in which you wish to include the constants as arguments to the function, *e.g.*, x(t, c_1, c_2) where c_1 and c_2 are the values of the function and its first derivative at $t = 1$. Then you would use

```
> XXX := unapply( X, (t, c1, c2) );
```

$$XXX := (t,\ c1,\ c2) \rightarrow$$
$$(2\,t\,e^{(2-2\,t)} - e^{(2-2\,t)})\,c1 + (t\,e^{(2-2\,t)} - e^{(2-2\,t)})\,c2 + \frac{1}{3}\,t^3\,e^{(-2\,t)} + \frac{2}{3}\,e^{(-2\,t)} - t\,e^{(-2\,t)}$$

and the resulting function, XXX, can be used to find the solution passing through the point (1, 0) with unit slope as follows

```
> XXX(t,0,1);
```

$$\frac{1}{3}\,t^3\,e^{(-2\,t)} + \frac{2}{3}\,e^{(-2\,t)} - e^{(2-2\,t)} - t\,e^{(-2\,t)} + t\,e^{(2-2\,t)}$$

The same computation using the other two forms of the solution, X and XX, would appear as:

```
> subs( c1=0, c2=1, X );
```

$$\frac{1}{3}\,t^3\,e^{(-2\,t)} + \frac{2}{3}\,e^{(-2\,t)} - e^{(2-2\,t)} - t\,e^{(-2\,t)} + t\,e^{(2-2\,t)}$$

```
> subs( c1=0, c2=1, XX(t) );
```

$$\frac{1}{3}\,t^3\,e^{(-2\,t)} + \frac{2}{3}\,e^{(-2\,t)} - e^{(2-2\,t)} - t\,e^{(-2\,t)} + t\,e^{(2-2\,t)}$$

Next, consider the problem of verifying that the solution is correct.

```
> simplify( subs( x(t)=X, ODE ) );
```

$$2\,t\,e^{(-2\,t)} = 2\,t\,e^{(-2\,t)}$$

```
> simplify( subs( x(t)=XX(t), ODE ) );
```

$$2\,t\,e^{(-2\,t)} = 2\,t\,e^{(-2\,t)}$$

```
> simplify( subs( x(t)=XXX(t,c1,c2), ODE ) );
```

$$2\,t\,e^{(-2\,t)} = 2\,t\,e^{(-2\,t)}$$

Note that, because both x and XX are functions of a single variable, it is also possible to use simplify(subs(x=XX, ODE));. The same is not true for XXX, as this function requires three arguments.

To conclude this discussion, let's plot the solutions that have critical points (*i.e.*, c2=0) at (1,-1), (1,1), and (1,3) using each of X, XX, and XXX:

```
> S1 := { seq( subs( c1=C, c2=0, X ), C={-1,1,3} ) };
```

$$S1 := \{-2\,t\,e^{(2-2\,t)} + e^{(2-2\,t)} + \frac{1}{3}\,t^3\,e^{(-2\,t)} + \frac{2}{3}\,e^{(-2\,t)} - t\,e^{(-2\,t)},$$
$$6\,t\,e^{(2-2\,t)} - 3\,e^{(2-2\,t)} + \frac{1}{3}\,t^3\,e^{(-2\,t)} + \frac{2}{3}\,e^{(-2\,t)} - t\,e^{(-2\,t)},$$
$$2\,t\,e^{(2-2\,t)} - e^{(2-2\,t)} + \frac{1}{3}\,t^3\,e^{(-2\,t)} + \frac{2}{3}\,e^{(-2\,t)} - t\,e^{(-2\,t)}\}$$

```
> S2 := { seq( subs( c1=C, c2=0, XX('t') ), C={-1,1,3} ) };
```

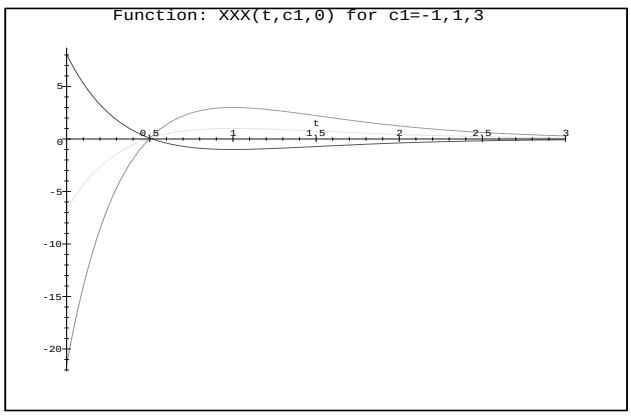
$$S2 := \{-2\,t\,e^{(2-2\,t)} + e^{(2-2\,t)} + \frac{1}{3}\,t^3\,e^{(-2\,t)} + \frac{2}{3}\,e^{(-2\,t)} - t\,e^{(-2\,t)},$$
$$6\,t\,e^{(2-2\,t)} - 3\,e^{(2-2\,t)} + \frac{1}{3}\,t^3\,e^{(-2\,t)} + \frac{2}{3}\,e^{(-2\,t)} - t\,e^{(-2\,t)},$$
$$2\,t\,e^{(2-2\,t)} - e^{(2-2\,t)} + \frac{1}{3}\,t^3\,e^{(-2\,t)} + \frac{2}{3}\,e^{(-2\,t)} - t\,e^{(-2\,t)}\}$$

```
> S3 := { seq( XXX('t',C,0), C={-1,1,3} ) };
```

$$S3 := \{-2\,t\,e^{(2-2\,t)} + e^{(2-2\,t)} + \frac{1}{3}\,t^3\,e^{(-2\,t)} + \frac{2}{3}\,e^{(-2\,t)} - t\,e^{(-2\,t)},$$
$$6\,t\,e^{(2-2\,t)} - 3\,e^{(2-2\,t)} + \frac{1}{3}\,t^3\,e^{(-2\,t)} + \frac{2}{3}\,e^{(-2\,t)} - t\,e^{(-2\,t)},$$
$$2\,t\,e^{(2-2\,t)} - e^{(2-2\,t)} + \frac{1}{3}\,t^3\,e^{(-2\,t)} + \frac{2}{3}\,e^{(-2\,t)} - t\,e^{(-2\,t)}\}$$

Note that the use of single quotes (') is needed so that the evaluation of t is delayed until after the subs command is completed. The same approach is needed to produce a plot of these solutions.

```
> plot( S3, 't'=0..3, title=`Function:  XXX(t,c1,0) for c1=-1,1,3`
);
```



Do these functions possess the expected properties?

The structure of the solution to this equation – a linear combination of a basis of solutions to the homogeneous equation plus a particular solution – is evident the solutions we have found. The individual components can be easily extracted and manipulated. Here we demonstrate how a general solution is used to determine the solution of a specific initial-value problem, say x(1) = 3, *x'(1) = 0*.

The first step is to find the general solution:

```
> GSOLN := rhs( dsolve( ODE, x(t) ) );
```

$$GSOLN := \frac{1}{3}\,t^3\,e^{(-2\,t)} + _C1\,e^{(-2\,t)} + _C2\,t\,e^{(-2\,t)}$$

The two equations that must be satisfied by $_C1$ and $_C2$ can be constructed:

```
> eq1 := subs( t=1, GSOLN=3 );
```

```
> eq2 := subs( t=1, diff(GSOLN,t)=0 );
```

$$eq1 := \frac{1}{3}\,e^{(-2)} + _C1\,e^{(-2)} + _C2\,e^{(-2)} = 3$$

$$eq2 := \frac{1}{3}\,e^{(-2)} - 2\,_C1\,e^{(-2)} - _C2\,e^{(-2)} = 0$$

and solved

```
> solC := solve( { eq1, eq2 }, { _C1, _C2 } );
```

$$solC := \left\{_C2 = -\frac{e^{(-2)} - 6}{e^{(-2)}},\ _C1 = \frac{1}{3}\,\frac{2\,e^{(-2)} - 9}{e^{(-2)}}\right\}$$

These solutions obviously simplify:

```
> solC := simplify( solC );
```

$$solC := \{_C2 = -(e^{(-2)} - 6)\,e^2,\ _C1 = \frac{1}{3}\,(2\,e^{(-2)} - 9)\,e^2\}$$

Floating point approximations to these constants are obtained using fnormal or evalf:

```
> evalf( solC );
```

$$\{_C2 = 43.33433660,\ _C1 = -21.50050163\}$$

The solution to the IVP is found to be

```
> IVPsoln := factor( subs( solC, GSOLN ) );
```

$$IVPsoln := -\frac{1}{3}\,e^{(-2\,t)}\,(-t^3 - 2\,e^2\,e^{(-2)} + 9\,e^2 + 3\,e^2\,t\,e^{(-2)} - 18\,e^2\,t)$$

The validity of this solution is confirmed by comparison with our earlier results:

```
> simplify( IVPsoln - subs( c1=3, c2=0, X ) );
```

$$0$$

```
> simplify( IVPsoln - subs( c1=3, c2=0, XX(t) ) );
```

$$0$$

```
> simplify( IVPsoln - XXX(t,3,0) );
```

$$0$$

To conclude, let's see three different ways in which the terms contributing to the homogeneous and particular solutions can be obtained. Working directly from the general solution:

```
> Xp := subs( _C1=0, _C2=0, GSOLN );
```

$$Xp := \frac{1}{3}\, t^3\, e^{(-2\,t)}$$

```
> Xh := GSOLN - Xp;
```

$$Xh := _C1\, e^{(-2\,t)} + _C2\, t\, e^{(-2\,t)}$$

```
> X1 := subs( _C1=1, _C2=0, Xh );
```

$$X1 := e^{(-2\,t)}$$

```
> X2 := subs( _C1=0, _C2=1, Xh );
```

$$X2 := t\, e^{(-2\,t)}$$

Alternatively, the same information can be obtained by using dsolve to solve appropriate modifications of the original problem:

```
> ODEh := lhs(ODE)=0;
```

$$ODEh := (\frac{\partial^2}{\partial t^2}\,\mathrm{x}(t)) + 4\,(\frac{\partial}{\partial t}\,\mathrm{x}(t)) + 4\,\mathrm{x}(t) = 0$$

```
> IC1 := x(0)=0, D(x)(0)=1;
> X1 := rhs( dsolve( { ODEh, IC1 }, x(t) ) );
```

$$IC1 := \mathrm{x}(0) = 0,\ \mathrm{D}(x)(0) = 1$$

$$X1 := t\, e^{(-2\,t)}$$

```
> IC2 := x(0)=1, D(x)(0)=0;
> X2 := rhs( dsolve( { ODEh, IC2 }, x(t) ) );
```

$$IC2 := \mathrm{x}(0) = 1,\ \mathrm{D}(x)(0) = 0$$

$$X2 := e^{(-2\,t)} + 2\,t\,e^{(-2\,t)}$$

```
> ICp := x(0)=0, D(x)(0)=0;
> Xp := rhs( dsolve( { ODE, ICp }, x(t) ) );
```

$$ICp := \mathrm{x}(0) = 0,\ \mathrm{D}(x)(0) = 0$$

$$Xp := \frac{1}{3}\,t^3\,e^{(-2\,t)}$$

A third option is to use the optional argument, output=basis, for the dsolve command (see ?dsolve):

```
> BASIS := dsolve( ODE, x(t), output=basis );
```

$$BASIS := [[e^{(-2\,t)},\ t\,e^{(-2\,t)}],\ \frac{1}{3}\,t^3\,e^{(-2\,t)}]$$

```
> XX1 := BASIS[1][1];
```

$$XX1 := e^{(-2\,t)}$$

```
> XX2 := BASIS[1][2];
```

$$XX2 := t\,e^{(-2\,t)}$$

```
> XXp := BASIS[2];
```

$$XXp := \frac{1}{3}\,t^3\,e^{(-2\,t)}$$

Although, to be honest, in this case it is probably simplest to explicitly identify the appropriate terms in the general solution and explicitly define the components of the solution.

```
> x1 := exp(-2*t);
> x2 := t*exp(-2*t);
> xp := t^3/3 * exp(-2*t);
```

$$x1 := e^{(-2\,t)}$$

$$x2 := t\,e^{(-2\,t)}$$

$$xp := \frac{1}{3}\,t^3\,e^{(-2\,t)}$$

This example hopefully illustrates that almost all manipulations can be done in Maple, some steps should still be done by hand.

Additional information about dsolve is available from the on-line help (?dsolve). Note, in particular, that dsolve may return an implicit solution or a solution in parametric form. Explicit solutions can, sometimes, be coerced by using the optional argument explicit=true. Other optional arguments are discussed later in this supplement.

5 Systems of ODEs

Systems of ODEs are analyzed in a manner that parallels the previous examples for a single equation. The system of equations is specified as a set (with or without initial conditions); the dependent variables are also specified as a set. To illustrate, consider the (linearized) model of a pendulum: θ"$(t) + 3\,\theta(t) = 0$. This second-order equation is equivalent to the system of first-order equations

```
> SYS := { diff( theta(t), t ) = v(t), diff( v(t), t ) = -3*theta(t)
};
```

$$SYS := \{\frac{\partial}{\partial t}\,\mathrm{v}(t) = -3\,\theta(t),\ \frac{\partial}{\partial t}\,\theta(t) = \mathrm{v}(t)\}$$

in terms of the dependent variables

```
> FNS := { theta(t), v(t) };
```

$$FNS := \{\theta(t),\ \mathrm{v}(t)\}$$

The general solution is found to be

```
> SOL := dsolve( SYS, FNS );
```

$$SOL :=$$

$$\{\mathrm{v}(t) = \cos(\sqrt{3}\,t)\,_C1 - \sqrt{3}\sin(\sqrt{3}\,t)\,_C2,\ \theta(t) = \frac{1}{3}\,\sqrt{3}\sin(\sqrt{3}\,t)\,_C1 + \cos(\sqrt{3}\,t)\,_C2\}$$

We are now in a position to understand why it is important that dsolve returns (a set of) equations. Because a set is unordered, there is no reason to expect the elements of the solution set to appear in the same order each time Maple is executed. That is, there is no "first" term in the solution vector. There is, nonetheless, a very simple means to ensure that the appropriate term from the solution is extracted. It is still possible to obtain the individual component as either an expression

```
> SOLV := subs( SOL, v(t) );
```

$$SOLV := \cos(\sqrt{3}\,t)\,_C1 - \sqrt{3}\sin(\sqrt{3}\,t)\,_C2$$

or as a function

```
> SOLT := unapply( subs( SOL, theta(t) ), t );
```

$$SOLT := t \to \frac{1}{3}\sqrt{3}\sin(\sqrt{3}\,t)\,_C1 + \cos(\sqrt{3}\,t)\,_C2$$

Most systems of ODEs do not have explicit solutions. Maple has no difficulties producing series solutions for systems. We illustrate using the corresponding nonlinear pendulum equation

```
> SYSnl := { diff( theta(t), t ) = v(t), diff( v(t), t ) = -3*sin(
theta(t) ) };
```

$$SYSnl := \{\frac{\partial}{\partial t}\,\mathrm{v}(t) = -3\sin(\theta(t)),\ \frac{\partial}{\partial t}\,\theta(t) = \mathrm{v}(t)\}$$

Only a small number of terms will be needed to compare with the linearized solution.

```
> Order := 3:
```

```
> SOLnl := dsolve( SYSnl, FNS, type=series );
```

$$SOLnl := \{\mathrm{v}(t) = \mathrm{v}(0) - 3\sin(\theta(0))\,t - \frac{3}{2}\cos(\theta(0))\,\mathrm{v}(0)\,t^2 + \mathrm{O}(t^3),$$
$$\theta(t) = \theta(0) + \mathrm{v}(0)\,t - \frac{3}{2}\sin(\theta(0))\,t^2 + \mathrm{O}(t^3)\}$$

```
> SOLnlV := subs( SOLnl, v(t) );
```

$$SOLnlV := \mathrm{v}(0) - 3\sin(\theta(0))\,t - \frac{3}{2}\cos(\theta(0))\,\mathrm{v}(0)\,t^2 + \mathrm{O}(t^3)$$

Comparisons with the solution to the linearized equation can be performed using the series command, as follows:

```
> series( SOLV, t=0 );
```

$$_C1 - 3\,_C2\,t - \frac{3}{2}\,_C1\,t^2 + \mathrm{O}(t^3)$$

The real power of Maple for systems of ODEs is seen in its handling of numerical solutions. This will be demonstrated in the next section.

6 Numeric Solutions

Numeric solutions to initial-value problems can be used in a variety of ways in an introductory course, including: direction fields, qualitative behavior of solutions, existence theory, and regularity theory. The dsolve command can be used for each of these topics (see ?dsolve,numeric). The default method used is rkf45, a Maple implementation of the Fehlberg fourth-fifth order Runge-Kutta method (see subroutine RKF45 appearing in COMPUTER METHODS FOR MATHEMATICAL COMPUTATIONS by G.E. Forsythe, M.A. Malcolm, and C.B. Moler); a second algorithm is a seventh-eighth order continuous Runge-Kutta method, dverk78.

6.1 New Numerical Methods in Release 4

Release 4 offers an expanded list of built-in numerical methods that can be used with dsolve. For a full listing of available methods, see ?dsolve,numeric and ?dsolve,classical.

To begin our discussion, consider the initial value problem

```
> ODE := diff( x(t), t ) = x(t)^2 + t:
> IC := x(0)=0:
> IVP := { ODE, IC };
```

$$IVP := \{\frac{\partial}{\partial t}\mathrm{x}(t) = \mathrm{x}(t)^2 + t,\ \mathrm{x}(0) = 0\}$$

While this problem does have an explicit solution, it cannot be found using methods typically found in an introductory course. But, it is reasonable to ask, *e.g.*, if a solution does exist for all t>0. Numerical solutions can be used, with caution, to discuss questions of this type.

The inclusion of the optional argument type=numeric is the basic interface to the numerical version of dsolve.

```
> numSOLN := dsolve( IVP, x(t), type=numeric );
```

$$numSOLN := \mathbf{proc}(rkf45_x) \ldots \mathbf{end}$$

The output from dsolve is quite different. What this is saying is that numSOLN is a Maple procedure that can be used to compute the solution at a given (numeric) value of t. For example:

```
> numSOLN( 0 );
```

$$[t = 0,\ \mathrm{x}(t) = 0]$$

```
> numSOLN( 1/10 );
```

$$[t = \frac{1}{10}, \mathrm{x}(t) = .005000500074847118]$$

Notice that the output from the procedure created by dsolve is a list of equations; one equation for the independent variable and one equation for each dependent variable. The structure of this solution is ideal for use with the subs command to extract specific components of a solution, or even to use the results in subsequent calculations. But first, let's evaluate the solution at more points

```
> numSOLN( 1.25 );
```

$$[t = 1.25, \mathrm{x}(t) = .9834238111018938]$$

```
> numSOLN( 1 );
```

$$[t = 1, \mathrm{x}(t) = .5571618043303169]$$

```
> numSOLN( 0 );
```

$$[t = 0, \mathrm{x}(t) = 0]$$

```
> numSOLN( 1 );
```

$$[t = 1, \mathrm{x}(t) = .5571617684443275]$$

These last result is a little distressing – why has the value of the solution changed from the first time the solution was evaluated? To explain this, it is necessary to realize that each call to numSOLN uses the closer of the initial condition and the most recent evaluation as the initial conditions for the current call. Thus, the first evaluation of the solution at $t = 1$ was determined using the values returned from the call with $t = 1.25$. The evaluation at $t = 0$ requires no calculations – it simply uses the initial condition. The second evaluation at $t = 1$ is fulfilled by integrating from $t = 0$ to $t = 1$. The difference between the two values gives an indication of the accumulation error for this solution. To reset the solution to the original initial condition, include startinit=true as an optional argument (see ?dsolve,numeric) or simply re-execute the dsolve command. This design characteristic can take some time to adjust to, but is quite reasonable for most uses of a numerical solution.

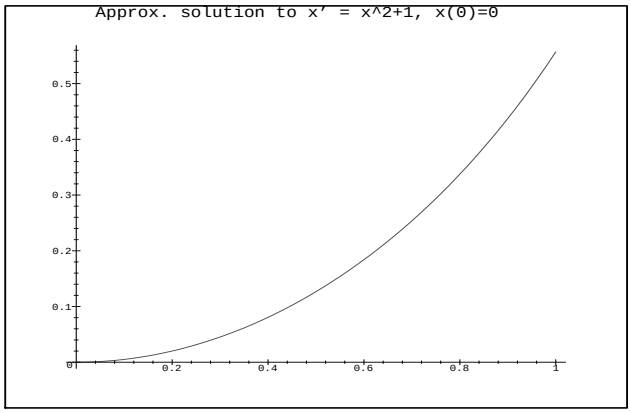
6.2 Update for Release 4

The choice of initial condition for evaluating a numerical solution is new to Release 4. Previous releases always used the last point at which the function was evaluated. The startinit=true optional argument is also new.

The plot of a numerical solution can be created in a number of different ways. One of the simplest is to use the odeplot command from the plots package,

which must be loaded prior to use. The basic syntax for odeplot expects the first argument to be the output from numeric dsolve, the second argument should be a list of two or three expressions involving the dependent and independent variables. For example,

```
> numSOLN := dsolve( IVP, x(t), type=numeric ):
> odeplot( numSOLN, [ t, x(t) ], 0..1,
> title=‘Approx.  solution to x’ = x^2+1, x(0)=0‘ );
```



Both numeric dsolve and odeplot work equally well with higher-order equations and with systems. The basic operations for higher-order problem will be illustrated using the damped nonlinear pendulum.

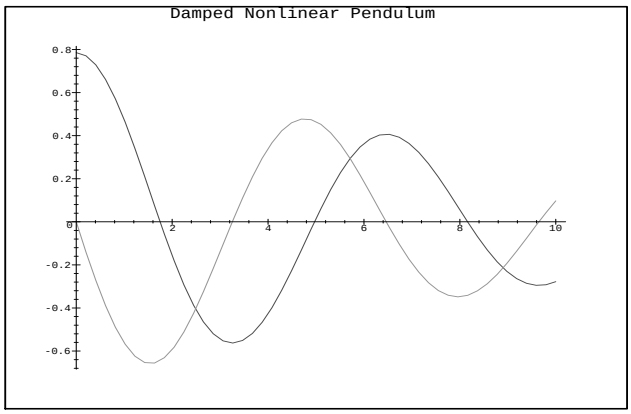
```
> ODE := diff( theta(t), t$2 ) + 0.2*diff( theta(t), t ) + sin(theta(t))
= 0:
> IC := theta(0)=Pi/4, D(theta)(0)=0:
> IVP := { ODE, IC };
```

$$IVP := \{(\frac{\partial^2}{\partial t^2}\,\theta(t)) + .2\,(\frac{\partial}{\partial t}\,\theta(t)) + \sin(\theta(t)) = 0,\ \theta(0) = \frac{1}{4}\,\pi,\ \mathrm{D}(\theta)(0) = 0\}$$

```
> PEND := dsolve( IVP, theta(t), type=numeric ):
```

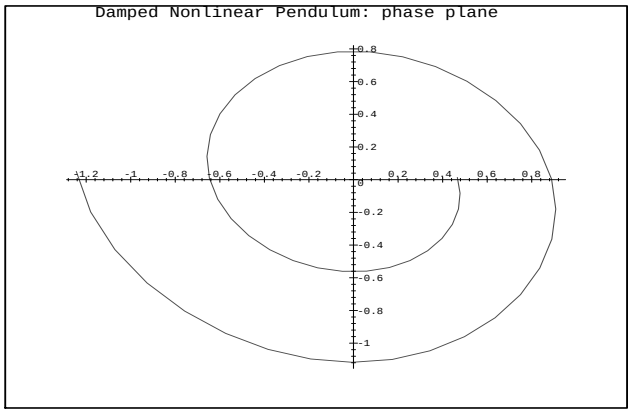
Multiple graphs are included in one plot when the second argument to odeplot is a list of lists. (Note how the derivative of the solution is selected using diff.)

```
> odeplot( PEND, [ [ t, theta(t) ], [ t, diff(theta(t),t) ] ],
0..10,
> title=‘Damped Nonlinear Pendulum‘ );
```



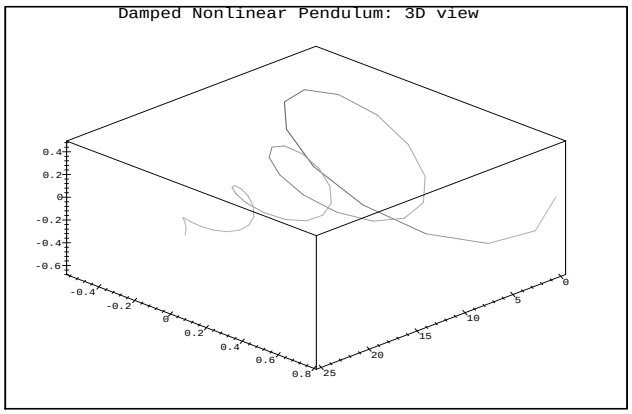
The phase portrait is also easy to create

```
> odeplot( PEND, [ diff(theta(t),t), theta(t) ], -5..5,
> title=‘Damped Nonlinear Pendulum:  phase plane‘ );
```



A three-dimensional view can sometimes be quite illustrative.

```
> odeplot( PEND, [ t, theta(t), diff(theta(t),t) ], 0..25,
> title=‘Damped Nonlinear Pendulum:  3D view‘, axes=BOXED );
```

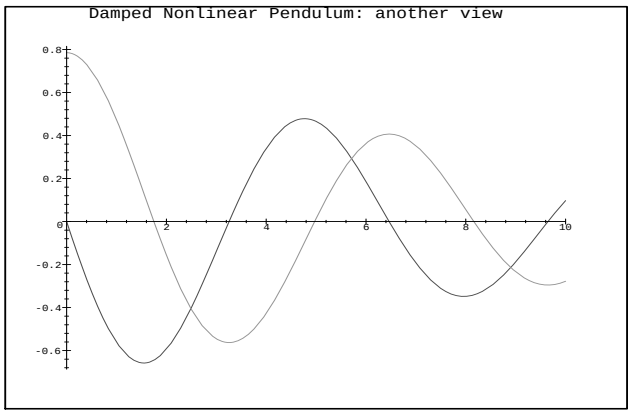


There are times when the standard output from numeric dsolve is not suitable for further processing. The output= and value= optional arguments provide some flexibility. Specifying output=listprocedure will cause dsolve to return a Maple list of procedures. (As has been used previously, the default, which can be specified using output=procedurelist, returns a single procedure whose values are lists of equations.)

```
> PEND2 := dsolve( IVP, theta(t), type=numeric, output=listprocedure
);
```

$$PEND2 := [t = (\mathbf{proc}(t) \ldots \mathbf{end}), \theta(t) = (\mathbf{proc}(t) \ldots \mathbf{end}), \frac{\partial}{\partial t}\theta(t) = (\mathbf{proc}(t) \ldots \mathbf{end})]$$

```
> THETA := subs( PEND2, theta(t) ):
> dTHETA := subs( PEND2, diff(theta(t),t) ):
> plot( { THETA, dTHETA }, 0..10,
> title=`Damped Nonlinear Pendulum:  another view` );
```



The value= argument is used when the value of the solution is needed only for specified values of the the independent variable. This can be particularly useful when creating a table of data:

```
> VALUES := array([ i/100 $ i=320..330 ]);
```

$$VALUES := \left[\frac{16}{5}, \frac{321}{100}, \frac{161}{50}, \frac{323}{100}, \frac{81}{25}, \frac{13}{4}, \frac{163}{50}, \frac{327}{100}, \frac{82}{25}, \frac{329}{100}, \frac{33}{10}\right]$$

```
> PEND3 := dsolve( IVP, theta(t), type=numeric, value= VALUES );
```

$$PEND3 := \left[\begin{array}{c} \left[t,\ \theta(t),\ \frac{\partial}{\partial t}\theta(t)\right] \\ \left[\begin{array}{lll} \frac{16}{5} & -.5620403790 & -.02994513342 \\ \frac{321}{100} & -.5623128991 & -.02456030038 \\ \frac{161}{50} & -.5625316139 & -.01918415046 \\ \frac{323}{100} & -.5626966125 & -.01381712091 \\ \frac{81}{25} & -.5628079882 & -.008459647176 \\ \frac{13}{4} & -.5628658387 & -.003112162912 \\ \frac{163}{50} & -.5628702662 & .002224900014 \\ \frac{327}{100} & -.5628213769 & .007551111503 \\ \frac{82}{25} & -.5627192815 & .01286604321 \\ \frac{329}{100} & -.5625640950 & .01816926855 \\ \frac{33}{10} & -.5623559366 & .02346036265 \end{array} \right] \end{array} \right]$$

Extracting information from this table can be frustrating. Note that the structure of PEND3 is a two-dimensional (column) array. The first element of PEND3 is the three-dimensional (row) vector which describes the contents of the matrix contained in the second element of PEND3.

```
> VARS := evalm( PEND3[1,1] );
```

$$VARS := \left[t,\ \theta(t),\ \frac{\partial}{\partial t}\theta(t)\right]$$

```
> VARS[2];
```

$$\theta(t)$$

It is also possible to nest the indices to directly access entries in this data structure. For example, the pendulum is at rest sometime between $t = 3.25$ and $t = 3.26$. At this time the angle describing the pendulum's position is approximately

```
> rest := ( PEND3[2,1][6,2] + PEND3[2,1][7,2] )/2;
```

$$rest := -.5628680525$$

which, converted to degrees, is

```
> evalf( rest * 180/Pi );
```

$$-32.24996384$$

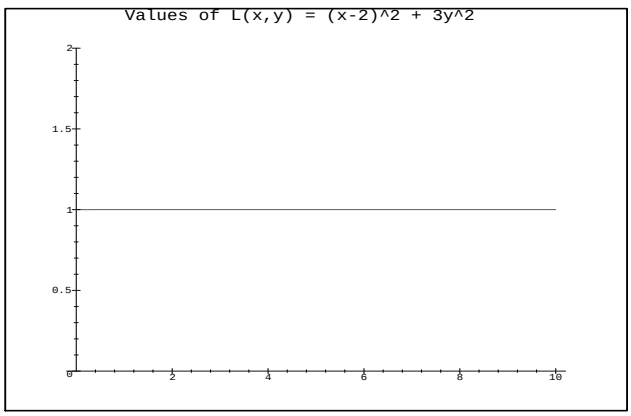
To illustrate the use of numeric dsolve for a system, consider the second-order system

```
> ODE := diff( x(t), t ) = 3*x(t)^2*y(t) - 6*y(t)^2,
> diff( y(t), t ) = - x(t)^3 + 2*x(t)^2 + 2*x(t)*y(t) - 4*y(t):
> IC := x(0) = 1, y(0)=0:
> IVP := { ODE, IC };
```

$$IVP := \{\mathrm{x}(0) = 1,\ \frac{\partial}{\partial t}\,\mathrm{x}(t) = 3\,\mathrm{x}(t)^2\,\mathrm{y}(t) - 6\,\mathrm{y}(t)^2,$$
$$\frac{\partial}{\partial t}\,\mathrm{y}(t) = -\mathrm{x}(t)^3 + 2\,\mathrm{x}(t)^2 + 2\,\mathrm{x}(t)\,\mathrm{y}(t) - 4\,\mathrm{y}(t),\ \mathrm{y}(0) = 0\}$$

The level curves of the (Liapunov) function $\mathrm{L}(x,\,y) = (x-2)^2 + 3\,y^2$ are trajectories for this system. That is, $\mathrm{L}(\mathrm{x}(t),\,\mathrm{y}(t)) = \mathrm{L}(x_0,\,y_0)$ for all t>0. This can be illustrated using numeric dsolve and odeplot as follows:

```
> numSOLN := dsolve( IVP, [ x(t), y(t) ], type=numeric ):
> odeplot( numSOLN, [ t, (x(t)-2)^2 + 3*y(t)^2 ], 0..10, view=0..2,
> title=`Values of L(x,y) = (x-2)^2 + 3y^2` );
```



Note that if the vertical range, *i.e.*, view=0..2, is omitted, then the plot looks a little surprising at first glance. A close examination of the default vertical range selected by Maple explains this apparent anomaly.

The preceding examples illustrate a few of the options that are available in numeric dsolve. Full details are available from the on-line help pages dsolve, dsolve[numeric], dsolve[dverk78], and plots[odeplot].

7 Series Solutions

Some differential equations, and systems, are most approachable using power series techniques. Maple provides access to this method by way of the type=series optional argument to dsolve.

Consider the differential equation, in normal form,

```
> ODE := diff( y(x), x$2 ) + 1/x * diff( y(x), x ) - 1/(x^2+1)
* y(x) = 0;
```

$$ODE := (\frac{\partial^2}{\partial x^2}\,\mathrm{y}(x)) + \frac{\frac{\partial}{\partial x}\,\mathrm{y}(x)}{x} - \frac{\mathrm{y}(x)}{x^2+1} = 0$$

The first few terms of the power series, centered at $x = 0$, can be obtained with the command

```
> dsolve( ODE, y(x), type=series );
```

$$\mathrm{y}(x) = _C1\,(1+\frac{1}{4}\,x^2+\mathrm{O}(x^3))+_C2\,(\ln(x)\,(1+\frac{1}{4}\,x^2+\mathrm{O}(x^3))+(-\frac{1}{4}\,x^2+\mathrm{O}(x^3)))$$

To obtain additional terms in the sequence, modify the system constant Order

```
> Order := 10:
> Y9 := dsolve( ODE, y(x), type=series );
```

$$\begin{aligned} Y9 := \mathrm{y}(x) = {} & _C1\,(1+\frac{1}{4}\,x^2-\frac{3}{64}\,x^4+\frac{5}{256}\,x^6-\frac{175}{16384}\,x^8+\mathrm{O}(x^{10}))+_C2(\\ & \ln(x)\,(1+\frac{1}{4}\,x^2-\frac{3}{64}\,x^4+\frac{5}{256}\,x^6-\frac{175}{16384}\,x^8+\mathrm{O}(x^{10})) \\ & +(-\frac{1}{4}\,x^2+\frac{1}{128}\,x^4+\frac{1}{1536}\,x^6-\frac{265}{196608}\,x^8+\mathrm{O}(x^{10}))) \end{aligned}$$

Note that $x = 0$ is a regular singular point for this equation. Note also the structure of this solution. The presence of the logarithmic factor is enlightening; that same term also complicates the conversion of this solution from an equation whose RHS is of type series into, *e.g.*, an expression that can be plotted.

Series solutions centered at other points can be requested by specifying a full set of initial conditions at the desired value of the independent variable. Here, the series expansion of the solution centered at $x = -1$ will be produced

```
> IVP := { ODE, y(-1)=c1, D(y)(-1)=c2 };
```

$$IVP := \left\{ \left(\frac{\partial^2}{\partial x^2}\,\mathrm{y}(x)\right) + \frac{\frac{\partial}{\partial x}\,\mathrm{y}(x)}{x} - \frac{\mathrm{y}(x)}{x^2+1} = 0,\ \mathrm{y}(-1) = c1,\ \mathrm{D}(y)(-1) = c2 \right\}$$

```
> Order:=6:
> Y5 := dsolve( IVP, y(x), type=series );
```

$$Y5 := \mathrm{y}(x) = c1 + c2\,(x+1) + \left(\frac{1}{4}\,c1 + \frac{1}{2}\,c2\right)(x+1)^2 + \left(\frac{5}{12}\,c2 + \frac{1}{6}\,c1\right)(x+1)^3 + \left(\frac{11}{96}\,c1 + \frac{1}{3}\,c2\right)(x+1)^4 + \left(\frac{1}{12}\,c1 + \frac{127}{480}\,c2\right)(x+1)^5 + \mathrm{O}((x+1)^6)$$

Here are two ways of obtaining the series solutions for two linearly independent solutions to this ODE. First, it is possible to simply specify two different sets of values for the constants c1 and c2:

```
> Y5a := subs( c1=1, c2=0, rhs(Y5 ) );
> Y5b := subs( c1=0, c2=1, rhs(Y5 ) );
```

$$Y5a := 1 + \frac{1}{4}\,(x+1)^2 + \frac{1}{6}\,(x+1)^3 + \frac{11}{96}\,(x+1)^4 + \frac{1}{12}\,(x+1)^5 + \mathrm{O}((x+1)^6)$$

$$Y5b := x + 1 + \frac{1}{2}\,(x+1)^2 + \frac{5}{12}\,(x+1)^3 + \frac{1}{3}\,(x+1)^4 + \frac{127}{480}\,(x+1)^5 + \mathrm{O}((x+1)^6)$$

Alternatively, Maple can collect the terms involving c1 and c2 separately. But, before this collect can be used, the solution must be converted to a polynomial. (So long as the O term is present, the solution is an equation whose RHS is of type series.)

```
> Y5 := convert( rhs(Y5), polynom );
```

$$Y5 := c1 + c2\,(x+1) + \left(\frac{1}{4}\,c1 + \frac{1}{2}\,c2\right)(x+1)^2 + \left(\frac{5}{12}\,c2 + \frac{1}{6}\,c1\right)(x+1)^3 + \left(\frac{11}{96}\,c1 + \frac{1}{3}\,c2\right)(x+1)^4 + \left(\frac{1}{12}\,c1 + \frac{127}{480}\,c2\right)(x+1)^5$$

```
> Y5 := collect( Y5, [ c1, c2 ] );
```

$$Y5 := \left(\frac{1}{4}\,(x+1)^2 + \frac{1}{6}\,(x+1)^3 + \frac{11}{96}\,(x+1)^4 + 1 + \frac{1}{12}\,(x+1)^5\right) c1 + \left(\frac{5}{12}\,(x+1)^3 + x + 1 + \frac{1}{2}\,(x+1)^2 + \frac{127}{480}\,(x+1)^5 + \frac{1}{3}\,(x+1)^4\right) c2$$

```
> Y5aa := subs( c1=1, c2=0, Y5 );
```

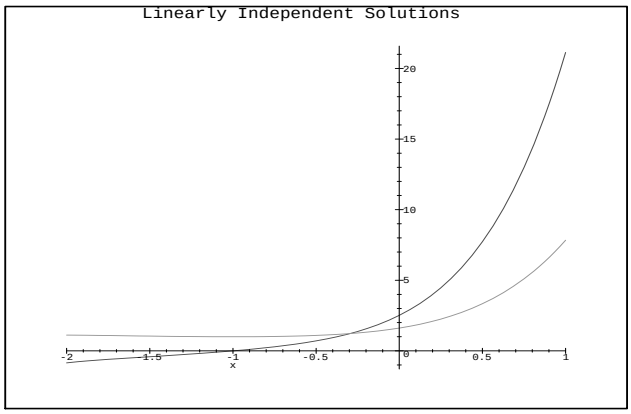
$$Y5aa := \frac{1}{4}\,(x+1)^2 + \frac{1}{6}\,(x+1)^3 + \frac{11}{96}\,(x+1)^4 + 1 + \frac{1}{12}\,(x+1)^5$$

```
> Y5bb := subs( c1=0, c2=1, Y5 );
```

$$Y5bb := \frac{5}{12}(x+1)^3 + x + 1 + \frac{1}{2}(x+1)^2 + \frac{127}{480}(x+1)^5 + \frac{1}{3}(x+1)^4$$

Note that the ordering of the terms in this last expression is less than optimal. (Controlling the order of terms in a polynomial is not a simple task in Maple.)

```
> plot( map( convert, { Y5a, Y5b }, polynom ), x=-2..1,
> title=`Linearly Independent Solutions` );
```



A third option would appear to possible using the output=basis as an optional argument. Unfortunately, this option does not work with type=series. For additional assistance, please consult the on-line help ?dsolve.

8 Laplace Transforms

All uses of Laplace transforms should begin by reading the integral transform (inttrans) library into the Maple session.

```
> with( inttrans );
```

[*addtable*, *fourier*, *fouriercos*, *fouriersin*, *hankel*, *hilbert*, *invfourier*, *invhilbert*, *invlaplace*, *laplace*, *mellin*]

8.1 Compatability with Release 3

The inttrans library combines several integral transformations into a single Maple library. This replaces readlib(laplace); that was used in earlier releases.

The two commands that form the Laplace transform pair are laplace and invlaplace. These commands can be used explicitly or implicitly to solve a differential equation. Both approaches will be illustrated.

Consider the the problem of determining the general solution to

```
> ODE := diff( y(t), t$2 ) + 3*diff( y(t), t ) - 4 * y(t) = 0;
```

$$ODE := (\frac{\partial^2}{\partial t^2}\,\mathrm{y}(t)) + 3\,(\frac{\partial}{\partial t}\,\mathrm{y}(t)) - 4\,\mathrm{y}(t) = 0$$

One approach is to begin by applying the Laplace transform to this equation and solving for the Laplace transform, $\mathrm{Y}(s)$, of the solution $\mathrm{y}(t)$:

```
> lapODE := laplace( ODE, t, s );
```

$$lapODE := s\,(s\,\mathrm{laplace}(\mathrm{y}(t),\,t,\,s) - \mathrm{y}(0)) - \mathrm{D}(y)(0) + 3\,s\,\mathrm{laplace}(\mathrm{y}(t),\,t,\,s) - 3\,\mathrm{y}(0) - 4\,\mathrm{laplace}(\mathrm{y}(t),\,t,\,s) = 0$$

The appearance of this equation is greatly improved with the use of an alias.

```
> alias( Y(s) = laplace( y(t), t, s ) ):
> lapODE;
```

$$s\,(s\,\mathrm{Y}(s) - \mathrm{y}(0)) - \mathrm{D}(y)(0) + 3\,s\,\mathrm{Y}(s) - 3\,\mathrm{y}(0) - 4\,\mathrm{Y}(s) = 0$$

The alias also makes it simpler to solve for the Laplace transform of y:

```
> SOLY := solve( lapODE, Y(s) );
```

$$SOLY := \frac{s\,\mathrm{y}(0) + \mathrm{D}(y)(0) + 3\,\mathrm{y}(0)}{s^2 + 3\,s - 4}$$

Next, the inverse Laplace transform is applied to find the solution $\mathrm{y}(t)$:

```
> SOLy := invlaplace( SOLY, s, t );
```

$$SOLy := -\frac{1}{5}\,e^{(-4\,t)}\,\mathrm{D}(y)(0) + \frac{1}{5}\,e^{(-4\,t)}\,\mathrm{y}(0) + \frac{4}{5}\,e^{t}\,\mathrm{y}(0) + \frac{1}{5}\,e^{t}\,\mathrm{D}(y)(0)$$

or, in a form that is sometimes more useful,

```
> collect( SOLy, [ y(0), D(y)(0) ] );
```

$$(\frac{1}{5}\,e^{(-4\,t)} + \frac{4}{5}\,e^{t})\,\mathrm{y}(0) + (-\frac{1}{5}\,e^{(-4\,t)} + \frac{1}{5}\,e^{t})\,\mathrm{D}(y)(0)$$

Alternatively, a one-step solution is possible with the use of the method=laplace optional argument to dsolve.

```
> dsolve( ODE, y(t), method=laplace );
```

$$\mathrm{y}(t) = -\frac{1}{5}\,e^{(-4\,t)}\,\mathrm{D}(y)(0) + \frac{1}{5}\,e^{(-4\,t)}\,\mathrm{y}(0) + \frac{4}{5}\,e^{t}\,\mathrm{y}(0) + \frac{1}{5}\,e^{t}\,\mathrm{D}(y)(0)$$

```
> collect( rhs("), [ y(0), D(y)(0) ] );
```

$$(\frac{1}{5}\,e^{(-4\,t)}+\frac{4}{5}\,e^{t})\,\mathrm{y}(0)+(-\frac{1}{5}\,e^{(-4\,t)}+\frac{1}{5}\,e^{t})\,\mathrm{D}(y)(0)$$

For additional assistance, please consult the on-line help: ?dsolve, as well as the help pages for the individual commands ?laplace, ?invlaplace

9 DESol

Maple provides a data structure for the explicit manipulation of a differential equation without attempting to solve the equation. This data structure, DESol, is not a prime candidate for use in an introductory course in differential equations, and is not discussed in any detail in this supplement. Full information is available from the on-line help: ?DESol

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demo-odes.mws – a Maple worksheet illustrating the use of Maple to solve and analyze ODEs and IVPs

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as part of the Maple Supplement for Nagle/Saff (Addison-Wesley)

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