**Math 142** **Lab Project #1** **Due Wed., Feb. 11 by 5:00 PM**

## Assignment

Your first lab project consists of questions (1), (3), and (4) from the back of this page. Each question is worth 10 points. The optional extra credit problem is worth 3 bonus points. If you prefer, you can use question (2) as a replacement for question (1), but you should clearly indicate if you want us to grade question (1) or question (2).

## Collaboration

You may work with others, but each student should submit a separately written project report. At the end of your report, you should list all of your collaborators. Simply copying another's work and calling it your own, or not giving credit to collaborators, is considered plagiarism and will be dealt with according to University regulations.

## What makes a good report?

Your final report for each project should have a clear and complete presentation of the project material. Each report will generally consist of paragraphs of text interspersed between various Maple commands, plots, and other output. All text should be written with complete sentences using proper grammar and correct spelling. A part of your grade will be based upon the clarity of your presentation. When proofreading your report, check to see if you are omitting important details or illustrative plots. Also check to see if you are including too many unnecessary details. Hand in work that you are proud to claim as your own.

In particular, do not simply turn in a Maple worksheet with a bunch of input and output. It is not the TA's job to figure out how your Maple output answers the various questions. It is your job to submit an easy-to-read project report. You may use pencil and paper, Microsoft Word with bits of output copied from Maple, Maple with text regions, or any other format which allows you to submit a clearly written report. Note also that you are not required to use Maple for all parts of your project. Some work is more easily done by hand.

## Submit project reports to your Lab TA (due Wed., Feb. 11 by 5:00 PM)

Sections 1 – 3 with Prof. Dix
Lab TA: Qi Wu
Contact Info: wuq@math.sc.edu, LC 107B, 576-5948, mailbox for Wu, Qi by LC 411

Sections 4 – 6 with Mr. Murphy
Lab TA: Elizabeth Perez
Contact Info: pereze@math.sc.edu, LC107B, 576-5948, mailbox for Perez by LC 411

Sections 7 – 9 with Prof. Bennett
Lab TA: Luke Owens
Contact Info: owensl@math.sc.edu, LC B004, 777-4674, mailbox for Owens, L by LC 411

# Introduction to Maple and Inverse Functions

**Questions**

(1) Let $f(x) = \ln(2 + \sin(x))$.
  (a) Find the largest interval containing $x = 0$ on which $f(x)$ is one-to-one.
  (b) Find the formula for the corresponding inverse function, $g(x) = f^{-1}(x)$.
  (c) Identify the domain of $f^{-1}(x)$.
  (d) Plot the function, its inverse, and the line $y = x$.
  (Remember to use different colors or thicknesses to distinguish the curves.)
  (e) For what values of $x$ is it true that $f(g(x)) = x$?
  (f) For what values of $x$ is it true that $g(f(x)) = x$?

(2) Let $f(x) = \dfrac{x}{x^2 - 2x + 4}$.
  (a) Find the three intervals on which the graph of $y = f(x)$ is one-to-one.
  (b) Let $g(x) = f^{-1}(x)$ denote the inverse function that is defined on the interval that contains $x = 0$.
  Find the function $g$. What is the domain of $g$?
  (c) Create a plot of $y = f(x)$ on its domain, the line $y = x$, and the portion of the graph of $y = g(x)$ that is the reflection of the graph of $y = f(x)$ across the line $y = x$.
  (For the third curve, the big question is how to determine a suitable domain.)
  (d) For what values of $x$ is it true that $f(g(x)) = x$?
  (e) For what values of $x$ is it true that $g(f(x)) = x$?

(3) Let $f(x) = \ln(x + \sqrt{x^2 - 1})$.
  (a) What is the domain of $f$?
  (b) Explain why $f$ has an inverse on its domain?
  (c) Find the simplest possible expression for the inverse of $f$.
  (d) What is $(f^{-1})'(x)$? (Be sure to write this answer in the simplest possible form.)
  (e) Compute, and simplify as much as possible, $(f^{-1})'(f(x))$ and $\frac{1}{f'(x)}$.
  (f) How should the expressions found in (e) be related? (Explain.)
  (g) Verify that the two expressions found in (e) are equivalent. (Note that another way to check that $a = b$, is to check that $a - b$ simplifies to 0.)

(4) A picture 5 feet in height is hung on a wall so that its bottom is 8 feet from the floor. A viewer with eye level at 66 inches stands $x$ feet from the wall. Let $\theta$ be the angle subtended by the picture at eye level.
  (a) Sketch the situation described in this problem.
  (b) What is $\theta$ when the viewer stands 10 feet from the wall?
  (Express your answer in both radians and degrees.)
  (c) Express $\theta$ as a function of $x$.
  (d) Create a plot of $\theta$ as a function of $x$.
  (e) Use calculus to find how far from the wall the viewer should stand to maximize the angle subtended by the picture at eye level.
  (f) What is the value of $\theta$ when the viewer stands at the location found in (e)?
  (Express your answer in both radians and degrees.)
  (g) If there is a low barricade 3 feet from the wall, and the room is 25 feet wide (measured perpendicular to the wall where the picture is mounted), what is the worst place to stand to view the picture? (Explain.)

Extra Credit Reconsider Question 4 when the height of the picture is $h$ feet, the bottom of the picture is $b$ feet above the floor, and the viewers's eye level is $a$ feet above the floor. Express the optimal location ($x$) and angle ($\theta$) in terms of $a$, $b$, and $h$. (Assume $a < b$.)