MATH 706 – Numerical Linear Algebra

Course Summary / Qualifying Exam Syllabus

Spring 2006

Professor Douglas Meade

1. Algorithms for Solving Linear Systems
    a. Triangular Systems
    b. Cholesky Decomposition (symmetric, positive definite)
    c. Gaussian Elimination (LU decomposition, partial pivoting)
    d. Operation counts

2. Floating-Point Computations
    a. Norms and condition number
    b. Propagation of roundoff errors
    c. Backward stability
    d. Convergence (relative / absolute)
    e. Residual analysis

3. Least Squares Problems
    a. Orthogonal matrices
    b. Normal equations
    c. Pseudo-inverse
    d. QR Decomposition (rotators and reflectors)
    e. Gram-Schmidt (and its variants)

4. Eigen Analysis
    a. Power Method (and its variants: inverse, shifted, Rayleigh quotients)
    b. Gershgorin Circle Theorem
    c. Similarity transformations
    d. Reduction to Hessenberg and tridiagonal forms
       (why not diagonal?)
    e. QR algorithm (for eigenvalues)
    f. Computing eigenvectors with QR algorithm
    g. Subspace iteration

5. Singular Value Decomposition
    a. Connection with Least Squares Problem
    b. Computation using the QR algorithm

6. Iterative Methods for Linear Systems
    a. Equivalence between linear systems and minimization problems
    b. Classical iterative methods (Jacobi, Gauss-Seidel)
    c. Descent methods (general and descent)
    d. Preconditioners
    e. Conjugate-gradient method