MATH 544 (Section 501)
Prof. Meade

Exam 1
18 February 2011

University of South Carolina
Spring 2011

Name: Key
SS # (last 4 digits): ____________

Instructions:

1. There are a total of 8 problems (not counting the Extra Credit problem) on 2 pages. Check that your copy of the exam has all of the problems.
2. No electronic or other inanimate objects can be used during this exam. All questions have been designed with this in mind and should not involve unreasonable manual calculations.
3. Be sure you answer the questions that are asked.
4. You must show all of your work to receive full credit for a correct answer. Correct answers with no supporting work will be eligible for at most half-credit.
5. Your answers must be written legibly in the space provided. You may use the back of a page for additional space; please indicate clearly when you do so.
6. Check your work. If I see *clear evidence* that you checked your answer (when possible) and you *clearly indicate* that your answer is incorrect, you will be eligible for more points than if you had not checked your work.

Problem	Points	Score
1	20	
2	15	
3	10	
4	15	
5	10	
6	10	
7	5	
8	15	
Total	100	

Have a Great Weekend!

1. (20 points) Identify each statement as either True or False. You do *not* have to justify your answer to receive credit for a correct answer, but some explanation might make you eligible for some partial credit for an incorrect answer.

   (a) __F__ Every matrix is row equivalent to a unique matrix in echelon form (T if reduced echelon)

   (b) __T__ If a system of linear equations has two different solutions, it must have an infinite number of solutions.

   (c) __T__ If a system $A\mathbf{x} = \mathbf{b}$ has more than one solution, then so does the homogeneous system $A\mathbf{x} = \mathbf{0}$.

   (d) __T__ If A is an $m \times n$ matrix and the equation $A\mathbf{x} = \mathbf{b}$ is consistent for every $\mathbf{b}$ in $\mathbf{R}^m$, then A has m pivot columns.

   (e) __T__ If 3×3 matrices A and B each have three pivot positions, then A can be transformed into B by elementary row operations. $A \sim I \sim B$ so $A \sim B$.

   (f) __F__ If A is an $m \times n$ matrix and the equation $A\mathbf{x} = \mathbf{b}$ has more than one solution, then the equation $A\mathbf{x} = \mathbf{c}$ can have a unique solution. (will be either no soln or ∞ # of soln.)

   (g) __F__ If none of the vectors in the set $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ in $\mathbf{R}^3$ is a multiple of the other vectors, then S is linearly independent. (only valid for 2 vectors)

   (h) __F__ In some cases, it is possible for four vectors to span $\mathbf{R}^5$. (must have at least 5 to have a P NOT in every row).

   (i) __F__ If $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are vectors in $\mathbf{R}^2$, then $\mathbf{w}$ is a linear combination of $\mathbf{u}$ and $\mathbf{v}$. (one vector is a linear comb. of the others, but not necessarily $\vec{w}$ in terms of $\vec{u}$ & $\vec{v}$.)

   (j) __T__ A linear transformation is a function.

2. (15 points) Determine whether each of the following formulas is True or False for all scalars a and b and all $n \times n$ matrices A, B, and C.

   (a) __T__ $A(B + C)D = ABD + ACD$

   (b) __T__ $(A + B)(A - B) = A^2 + BA - AB - B^2$

   (c) __T__ If $B = AA^T$, then $B^T = B$ $\quad B^T = (AA^T)^T = A^{TT}A^T = AA^T = B$

   (d) __F__ $(aA + bB)^{-1} = \frac{1}{a}A^{-1} + \frac{1}{b}B^{-1}$

   (e) __T__ If $AB = BA$ and if B is invertible, then $B^{-1}A = AB^{-1}$. $\quad B^{-1}AB = B^{-1}BA = A$
   $B^{-1}A = B^{-1}ABB^{-1} = AB^{-1}$

3. (10 points) Describe, in parametric vector form, all solutions to $A\mathbf{x} = \mathbf{0}$ where A is row equivalent to
$$\begin{bmatrix} 1 & 5 & 2 & -6 & 9 & 0 \\ 0 & 0 & 1 & -7 & 4 & -9 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{②+9③\to②} \begin{bmatrix} 1 & 5 & 2 & -6 & 9 & 0 \\ 0 & 0 & 1 & -7 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{①-2②\to①} \begin{bmatrix} 1 & 5 & 0 & 8 & 1 & 0 \\ 0 & 0 & 1 & -7 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$
$$\vec{x} = \begin{bmatrix} -5x_2 - 8x_4 - x_5 \\ x_2 \\ 7x_4 - 5x_5 \\ x_4 \\ x_5 \\ 0 \end{bmatrix} = x_2 \begin{bmatrix} -5 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -8 \\ 0 \\ 7 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -1 \\ 0 \\ -5 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

4. (15 points) Determine h and k such that the solution set of the system (i) is empty, (ii) contains a unique solution, and (iii) contains infinitely many solutions.
$$\begin{aligned} x_1 + 4x_2 &= k \\ 3x_1 + hx_2 &= 9. \end{aligned}$$
$$\begin{bmatrix} 1 & 4 & k \\ 3 & h & 9 \end{bmatrix} \to \begin{bmatrix} 1 & 4 & k \\ 0 & h-12 & 9-3k \end{bmatrix}.$$
(i) empty: $h - 12 = 0$, $9 - 3k \neq 0$: $h = 12$, $k \neq 3$
(ii) unique: $h - 12 \neq 0$ $\quad h \neq 12$
(iii) ∞ : $h - 12 = 0$, $9 - 3k = 0$
$h = 12$, $k = 3$

5. (10 points) List all possible echelon forms of a 3×4 matrix A whose columns span $\mathbf{R}^3$.

6. (10 points) Let A be a 4×3 matrix in which the third column is the sum of the first two columns. Find a nontrivial solution to $A\mathbf{x} = \mathbf{0}$. $A = [\vec{a}_1\ \vec{a}_2\ \vec{a}_3]$ with $\vec{a}_3 = \vec{a}_1 + \vec{a}_2$ so $\vec{a}_1 + \vec{a}_2 - \vec{a}_3 = \vec{0}$ or $A\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} = \vec{0}$.
so $\vec{x} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$.

7. (5 points) Let $T : \mathbf{R}^3 \to \mathbf{R}^3$ be the linear transformation that reflects each vector through the plane $x_3 = 0$. Find the standard matrix of T. $T(x_1, x_2, x_3) = (x_1, x_2, -x_3)$
so $T(\vec{x}) = A\vec{x}$
where $A = [T(\vec{e}_1)\ T(\vec{e}_2)\ T(\vec{e}_3)]$
$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$.

8. (15 points) Find the second column of the inverse of the matrix $A = \begin{bmatrix} 1 & 3 & 8 \\ 2 & 4 & 11 \\ 1 & 2 & 5 \end{bmatrix}$.
$$[A \mid \vec{e}_2] = \begin{bmatrix} 1 & 3 & 8 & 0 \\ 2 & 4 & 11 & 1 \\ 1 & 2 & 5 & 0 \end{bmatrix} \xrightarrow[③-①\to③]{②-2①\to②} \begin{bmatrix} 1 & 3 & 8 & 0 \\ 0 & -2 & -5 & 1 \\ 0 & -1 & -3 & 0 \end{bmatrix} \xrightarrow{②-2③\to③} \begin{bmatrix} 1 & 3 & 8 & 0 \\ 0 & -2 & -5 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$
$$\xrightarrow[②+5③\to②]{①-8③\to①} \begin{bmatrix} 1 & 3 & 0 & -8 \\ 0 & -2 & 0 & 6 \\ 0 & 0 & 1 & 1 \end{bmatrix} \xrightarrow{-\frac{1}{2}②} \begin{bmatrix} 1 & 3 & 0 & -8 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 1 \end{bmatrix} \xrightarrow{①-3②\to①} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$
2nd col of A^{-1}