MATH 544 | University of South Carolina
Prof. Meade | Spring 2000

Exam 1
February 15, 2000

Name: ____________________
SS #: ____________________

Instructions:

1. There are a total of 6 problems on 5 pages. Check that your copy of the exam has all of the problems.
2. You must show all of your work to receive credit for a correct answer.
3. Your answers must be written legibly in the space provided. You may use the back of a page for additional space; please indicate clearly when you do so.

Problem	Points	Score
1	20	
2	21	
3	25	
4	12	
5	10	
6	12	
Total	100	

Happy (Belated) Valentine's Day!

1. (20 points) Consider the following system of linear equations:

$$\begin{array}{rcrcrcrcl} x_1 & + & x_2 & - & 2x_3 & - & 2x_4 & = & 0 \\ & & x_2 & & & + & 3x_4 & = & 0 \\ -x_1 & - & 3x_2 & + & 2x_3 & - & 4x_4 & = & 0 \end{array}$$

(a) Use the algorithm developed in class to write the general solution in parametric form.

(b) Write a set of two or three vectors that spans the solution set found in (a).

2. (21 points) Let

$$A = \begin{bmatrix} -4 & 12 \\ 1 & -3 \\ -3 & 8 \end{bmatrix}, \qquad B = \begin{bmatrix} 2 & 7 & 0 \\ -4 & -6 & 5 \\ 6 & 13 & -3 \end{bmatrix}, \qquad C = \begin{bmatrix} 1 & 5 & -3 & 2 \\ 0 & 4 & -9 & 18 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

For each of the above matrices, determine whether its columns are linearly independent. Give a reason for your answer. (Use as few row operations as possible.)

3. (25 points) Let $T : \mathrm{R}^3 \to \mathrm{R}^3$ be a linear transformation such that

$$T(\mathbf{e}_1) = \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}, \qquad T(\mathbf{e}_2) = \begin{bmatrix} 2 \\ 3 \\ 6 \end{bmatrix}, \qquad T(\mathbf{e}_3) = \begin{bmatrix} 0 \\ -8 \\ 5 \end{bmatrix},$$

(a) Find $T(\begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix})$.

(b) Find the standard matrix of T.

(c) Determine if T maps R^3 onto R^3.

(d) Do you have enough information to determine if T is a one-to-one *without doing any additional computations*? If so, is T one-to-one? If not, what additional information would you need?

4. (12 points) Use the inverse of a matrix to solve the system:

$$\begin{array}{rcrcr} 5x_1 & - & 6x_2 & = & 1 \\ -7x_1 & + & 8x_2 & = & -3 \end{array}$$

5. (10 points) Assume A, B, C, and D are invertible $n \times n$ matrices. Solve the matrix equation $A(XB^{-1} + C) = D$ for X.

6. (12 points) Identify each statement as either True or False. You do *not* have to justify your answer.

   (a) ______ In some cases, it is possible for six vectors to span $\mathbb{R}^5$.

   (b) ______ If a matrix A is $n \times n$ and if the equation $A\mathbf{x} = \mathbf{b}$ has a solution for some $\mathbf{b}$, then the columns of A span $\mathbb{R}^n$.

   (c) ______ If a system of linear equations has two different solutions, then it has infinitely many solutions.

   (d) ______ Every matrix is row equivalent to a unique matrix in echelon form.