HW Solutions - §9.1

#4. $\frac{d\vec{x}}{dt} = \begin{pmatrix} 1 & -4 \\ 4 & -7 \end{pmatrix}\vec{x}$

(a) $r_1 = r_2 = -3$ $\vec{v}^{(1)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ (only lin. indep. e-vector)

(b) $p = tr(A) = 1 + -7 = -6 < 0$ $\Rightarrow$ asymptotically stable

$q = det(A) = (1)(-7) - 4(-4) = -7 + 16 = 9$

$\Delta = p^2 - 4q = (-6)^2 - 4(9) = 36 - 36 = 0$ $\Rightarrow$ proper or improper node

but, because there's only 1 e-vector, it is improper

#10. $\frac{d\vec{x}}{dt} = \begin{pmatrix} 1 & 2 \\ -5 & -1 \end{pmatrix}\vec{x}$

(a) $\lambda_{1,2} = \pm 3i$ $\vec{v}^{(1)} = \begin{pmatrix} -2 \\ 1 - 3i \end{pmatrix}$

(b) $p = tr A = 0$

$q = det A = -1 + 10 = 9 > 0$ } $\Rightarrow$ stable center

$\Delta = p^2 - 4q = 0 - 4(9) = -36 < 0$

#12. $\frac{d\vec{x}}{dt} = \begin{pmatrix} 2 & -\frac{5}{2} \\ \frac{9}{5} & -1 \end{pmatrix}\vec{x}$

(a) $\lambda_{1,2} = \frac{1}{2} \pm \frac{\sqrt{3}}{2}i$ $\vec{v}^{(1)} = \begin{pmatrix} 5 \\ 5 - \sqrt{3}i \end{pmatrix}$

(b) $p = tr A = 1$

$q = det A = -2 + \frac{9}{2} = \frac{5}{2}$

$\Delta = p^2 - 4q = 1 - 10 = -9 < 0$ $\Rightarrow$ unstable spiral point.