§1.2 HW Solutions

#4.b. $\frac{dy}{dt} = ay - b$. $\quad \left(y_e = \frac{b}{a}\right)$.

Let $\mathcal{I}(t) = y(t) - y_e$.

Then $\mathcal{I}' = y' - 0 = y' = ay - b = a(\mathcal{I} + y_e) - b = a\mathcal{I} + a\left(\frac{b}{a}\right) - b$

$= a\mathcal{I}$.

so $\mathcal{I}(t)$ is a solution to $\underline{\mathcal{I}' = a\mathcal{I}}$.

#7.b. $\frac{dp}{dt} = 0.5p - 450, \qquad p(0) = p_0$.

To find the solution to this separable DE:

$$\int \frac{dy}{0.5p - 450} = \int dt$$

This would be a little easier if the separation was done as

$$\frac{dy}{p - 900} = \frac{1}{2}\, dt$$

$\frac{1}{0.5} \ln|0.5p - 450| = t + C$.

~~Plug~~ $\ln|0.5p - 450| = \frac{1}{2}t + C$.

$0.5p - 450 = Ce^{t/2}$

Plug in the IC: $0.5p_0 - 450 = C$

$\cancel{0.5p_0 = C + 450}$

$\cancel{p_0 =}$

so $0.5p - 450 = (0.5p_0 - 450)e^{t/2}$

$0.5p = 450 + (0.5p_0 - 450)e^{t/2}$

$p(t) = 900 + (p_0 - 900)e^{t/2}$.

To find when the population becomes extinct, we solve $p(t) = 0$:

$900 + (p_0 - 900)e^{t/2} = 0$

$(p_0 - 900)e^{t/2} = -900$

$e^{t/2} = \frac{900}{900 - p_0}$

note how extra "−" are avoided.

if $p_0 \geq 900$ then the RHS is not positive, and there is no time when the population becomes extinct

$\frac{t}{2} = \ln \frac{900}{900 - p_0}$

$t = 2\ln \frac{900}{900 - p_0}$ is the time when the population becomes extinct when $p_0 < 900$.

(you might add $p_0 > 0$ on physical grounds)