Homework Solution

§4.5 #33 (p.262)

Solve the IVP $\quad y' + y = \begin{cases} 1 & t \le T \\ 0 & t > T \end{cases}, \quad y(0) = 0.$

Step 1: Find the solution for $t \le T$, i.e. solve $y' + y = 1$, $y(0) = 0$.

The complementary solution is $y_c = c_1 e^{-t}$.

A particular solution can be sought in the form: $Y = A$.

Then $Y' = 0$ and $Y' + Y = 0 + A = 1 \implies A = 1$.

A particular solution is $y_p = 1$.

The general solution is $y(t) = c_1 e^{-t} + 1$.

To satisfy the IC: $y(0) = c_1 + 1 = 0 \implies c_1 = -1$.

For $t \le T$ the solution is $y(t) = 1 - e^{-t}$.

Note that $y(T) = 1 - e^{-T}$.

Step 2: Find the solution for $t > T$; i.e. solve $y' + y = 0$, $y(T) = 1 - e^{-T}$.

This ODE is homogeneous, so the general solution is $y = c_2 e^{-t}$.

To match the IC (at $t = T$): $y(T) = c_2 e^{-T} = 1 - e^{-T}$

so $c_2 = e^{T} - 1$.

For $t > T$ the solution is $y(t) = (e^{T} - 1) e^{-t}$,

Step 3: Put the 2 solutions together:

$$y(t) = \begin{cases} 1 - e^{-t} & t \le T \\ (e^{T} - 1) e^{-t} & t > T. \end{cases}$$