Solutions - HW5

§2.7 #1. $y' = 3+t-y, \quad y(0)=1.$

a. h=0.1

i	t_i	y_i	m_i	y_{i+1}	Exact	Error
0	0.0000	1.0000	2.0000	1.2000	1.0000	0.0000
1	0.1000	1.2000	1.9000	1.3900	1.1952	0.0048
2	0.2000	1.3900	1.8100	1.5710	1.3813	0.0087
3	0.3000	1.5710	1.7290	1.7439	1.5592	0.0118
4	0.4000	1.7439	1.6561	1.9095	1.7297	0.0142

b: h=0.05

i	t_i	y_i	m_i	y_{i+1}	Exact	Error
0	0.0000	1.0000	2.0000	1.1000	1.0000	0.0000
1	0.0500	1.1000	1.9500	1.1975	1.0988	0.0012
2	0.1000	1.1975	1.9025	1.2926	1.1952	0.0023
3	0.1500	1.2926	1.8574	1.3855	1.2893	0.0033
4	0.2000	1.3855	1.8145	1.4762	1.3813	0.0042
5	0.2500	1.4762	1.7738	1.5649	1.4712	0.0050
6	0.3000	1.5649	1.7351	1.6517	1.5592	0.0057
7	0.3500	1.6517	1.6983	1.7366	1.6453	0.0064
8	0.4000	1.7366	1.6634	1.8198	1.7297	0.0069

about half the error as for $h=0.1$

d: This is a first-order DE, so we find an integrating factor.

$$\mu(y'+y) = (3+t)\mu$$

$$(\mu y)' = \mu y' + \mu' y \implies \mu' = \mu \quad \text{so } \mu = e^t.$$

$$\left[\begin{array}{ll} u = t & dv = e^t\,dt \\ du = dt & v = e^t \end{array}\right]$$

Next, $(e^t y)' = (3+t)e^t$

so $$e^t y = \int 3e^t + te^t\,dt = 3e^t + \int te^t\,dt = 3e^t + te^t - \int e^t\,dt$$

$$= 3e^t + te^t - e^t + C = 2e^t + te^t + C$$

$$\implies y = e^{-t}(2e^t + te^t + C) = 2+t+Ce^{-t}$$

For the initial condition: $1 = 2+0+C \implies C = -1$

and the exact solution to this IVP is $\underline{y = 2+t-e^{-t}}$

§2.9 #9. $y' = t^2+y^2, \quad y(0)=0.$

$$\varphi_0 = 0$$

$$\varphi_1 = \int_0^t s^2 + \varphi_0^2\,ds = \int_0^t s^2\,ds = \tfrac{1}{3}t^3.$$

$$\varphi_2 = \int_0^t s^2 + \varphi_1^2\,ds = \int_0^t s^2 + \tfrac{1}{9}s^6\,ds = \tfrac{1}{3}t^3 + \tfrac{1}{63}t^7$$

$$\varphi_3 = \int_0^t s^2 + \varphi_2^2\,ds = \int_0^t s^2 + \left(\tfrac{1}{9}s^6 + \tfrac{2}{189}s^{10} + \tfrac{1}{63^2}s^{14}\right)ds = \tfrac{1}{3}t^3 + \tfrac{1}{63}t^7 + \tfrac{2}{2079}t^{11} + \tfrac{1}{59535}t^{15}.$$