Math 520 (Section 001)
Prof. Meade

University of South Carolina
Fall 2011

Exam 3
November 22, 2011

Name: ____________________
SS # (last 4 digits): __________

Instructions:

1. There are a total of 6 problems on 2 pages. Check that your copy of the exam has all of the problems.
2. No electronic or other inanimate objects can be used during this exam. All questions have been designed with this in mind and should not involve unreasonable manual calculations.
3. Be sure you answer the questions that are asked.
4. You must show all of your work to receive full credit for a correct answer. Correct answers with no supporting work will be eligible for at most half-credit.
5. Your answers must be clearly labeled and written legibly on additional sheets of paper (that I will provide). Be sure each sheet contains your name and the work for each question is clearly labeled.
6. Check your work. If I see *clear evidence* that you checked your answer (when possible) and you *clearly indicate* that your answer is incorrect, you will be eligible for more points than if you had not checked your work.

Problem	Points	Score
1	30	
2	25	
3	10	
4	10	
5	10	
6	15	
Total	100	

Have a Happy Thanksgiving (and Beat Clemson!)

1. (30 points) This problem deals with finding a power series solution with $x_0 = 0$ for the differential equation $y'' - xy' - y = 0$.

   (a) Show that the recurrence relation is $a_{n+2} = \frac{1}{n+2} a_n$ for $n = 1, 2, \ldots$.
   What equation must be satisfied for $n = 0$?

   (b) Find the first four non-zero terms of the two solutions y_1 and y_2.

   (c) For what value of x is it easiest to compute $W[y_1, y_2](x)$? Compute this value of the Wronskian and state what information this provides about the solutions y_1 and y_2.

2. (25 points) Let $x^2y'' + (1 + x)y' + 3(\ln(x))y = 0$ with $y(1) = 2$ and $y'(1) = 0$.

   (a) Show that $y''(1) = 0$, $y'''(1) = -6$, and $y^{(4)}(1) = 42$.

   (b) Use the information in (a) to find the leading terms in the power series solution with $x_0 = 1$ for this initial value problem.

3. (10 points) Find the solution to $x^2y'' - 3xy' + 4y = 0$, $y(-1) = 2$, $y'(-1) = 3$.

4. (10 points) Transform $t^2u'' + tu' + (t^2 - 4)u = 0$ into a system of first-order equations.

5. (10 points) Let $A = \begin{pmatrix} 1 & 2 & 1 \\ -2 & 1 & 8 \\ 1 & -2 & -7 \end{pmatrix}$. Either compute the inverse of A or show that A is singular.

6. (15 points) Show that $\mathbf{x} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^t + 2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} te^t$ is a solution to $\mathbf{x}' = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^t$.